Basic Growth and Decay Practice Problems

1. Solve the following differential equations
   a. $\frac{dy}{dt} = -y$

   b. $\frac{dc}{dx} = -0.1c,\ c(0) = 20$

   c. $\frac{dz}{dt} = 3z,\ z(0) = 5$

2. The doubling time of a culture of bacteria is given to be 20 minutes. State:
   a. The appropriate differential equation that describes this growth

   b. The steady state and stability for the differential equation

   c. The appropriate initial conditions

   d. The exponential function (base e) that is the solution to that differential equation with the growth rate solved for. Use minutes for time t

3. The population $y(t)$ of a certain microorganism grows continuously and follows an exponential behaviour over time.
   a. Its doubling time is found to be 0.27 hours. What differential equation would you use to describe its growth?

   b. What is the growth rate and solution to the differential equation?

   c. With exposure to ultra-violet radiation, the population ceases to grow, and the microorganisms continuously die off. It is found that the half-life is then 0.1 hours. What differential equation would now describe the population?

   d. What is the decay rate and solution to this new differential equation?

4. The per capita birthrate of one species of rodent is 0.05 newborns per day. This means that, on average, each member of the population results in 5 newborn rodents every 100 days. Suppose that over the period of 1000 days there are no deaths, and that the initial population of rodents is 250.
   a. Write a differential equation for the population size at time t (in days).

   b. Write down the initial condition that it satisfies.

c. Find the solution, i.e. some function of time t that satisfies your differential equation and initial condition.

d. How many rodents are there after 1 year?

5. In general, when we have natural growth $\frac{dy}{dt} = ky$, we need to know $y(0) = y_0$ (the initial value) and $y(T) = y_1$ (some other amount at some other time). Determine k in terms of y_0, y_1, T.