SOLUTIONS for NATURAL GROWTH PART 1

1.
   a. $y = Ce^{-t}$
   b. $c = 20e^{-0.1x}$
   c. $z = 5e^{3t}$
2.
   a. $\frac{dB}{dt} = kB,\ k > 0$ where B is the bacteria population at time t (min)
   b. Steady state is $B = 0$ and is unstable
   c. $B(0) = B_0$ and $B(20) = 2B_0$
   d. $B = B_0 e^{kt}$ and solve for k using $t = 20 \Rightarrow k = \frac{\ln 2}{20} = 0.0346$ %/min
3.
   a. $\frac{dy}{dt} = ky,\ k > 0$
   b. $y = y_0 e^{kt}$ with $y(0.27) = 2y_0$, so $k = \frac{\ln 2}{0.27} = 2.567$ %/hour
   c. $\frac{dy}{dt} = cy,\ c < 0$
   d. $y = y_0 e^{ct}$ with $y(0.1) = 0.5y_0$, so $c = -6.931$ %/hour
4.
   a. $\frac{dR}{dt} = 0.05R$ where R is the rodent population
   b. $R(0) = 250$
   c. $R = 250e^{0.05t}$
   d. $R(365) = 21$ billion
5. The solution is $y = y_0 e^{kt}$ and we know $y_1 = y_0 e^{kT}$ so solve for $k = \frac{1}{T} \cdot \ln\left(\frac{y_1}{y_0}\right)$