# Series

Name

**1.** If the series $\sum_{n=1}^{\infty} a_n$ converges and $a_n > 0$ for all $n$, which of the following must be true?

(A) $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0$

(B) $|a_n| < 1$ for all $n$

(C) $\sum_{n=1}^{\infty} a_n = 0$

(D) $\sum_{n=1}^{\infty} na_n$ diverges

(E) $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges

**2.** Which of the following series diverge?

I. $\sum_{n=0}^{\infty} \left( \frac{\sin 2}{\pi} \right)^n$

II. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

III. $\sum_{n=1}^{\infty} \left( \frac{e^n}{e^n + 1} \right)$

(A) III only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III

## Series

**3.** If $f(x)=\sum_{k=1}^{\infty}(\sin^2 x)^k$, then $f(1)$ is

(A) 0.369

(B) 0.585

(C) 2.400

(D) 2.426

(E) 3.426

**4.** What is the value of $\sum_{n=1}^{\infty}\frac{(-3)^{n+1}}{5^n}$?

(A) $-\frac{15}{8}$

(B) $-\frac{9}{8}$

(C) $-\frac{3}{8}$

(D) $\frac{9}{8}$

(E) $\frac{15}{8}$

## Series

**5.** The $n\text{th}$ term test can be used to determine divergence for which of the following series?

1. $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k}\right)$
2. $\sum_{k=0}^{\infty} (-1)^k \left(\frac{k}{2k+1}\right)$
3. $\sum_{k=1}^{\infty} \frac{3k^2 - k^3}{5k^3}$

(A) III only

(B) I and III only

(C) II and III only

(D) I, II, and III

**6.** Let f be a positive, continuous, decreasing function. If $\int_1^{\infty} f(x)\,dx = 5$, which of the following statements about the series $\sum_{n=1}^{\infty} f(n)$ must be true?

(A) $\sum_{n=1}^{\infty} f(n) = 0$

(B) $\sum_{n=1}^{\infty} f(n)$ converges, and $\sum_{n=1}^{\infty} f(n) < 5$

(C) $\sum_{n=1}^{\infty} f(n) = 5$

(D) $\sum_{n=1}^{\infty} f(n)$ converges, and $\sum_{n=1}^{\infty} f(n) > 5$

(E) $\sum_{n=1}^{\infty} f(n)$ diverges

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**7.** The integral test can be used to determine that which of the following statements about the infinite series $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$ is true?

(A) The series converges because $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2} \square x = -1 + e$.

(B) The series converges because $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2} \square x = e$.

(C) The series converges because $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2} \square x = 1 - e$.

(D) The series diverges because $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2} \square x$ is not finite.

Consider the series $\sum_{n=2}^{\infty} \frac{1}{n^p \ln(n)}$, where $p \geq 0$.

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**8.** Determine whether the series converges or diverges for $p=1$ . Show your analysis.

Please respond on separate paper, following directions from your teacher.

**9.** Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

II. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

III. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

(A) None

(B) II only

(C) III only

(D) I and II only

(E) II and III only

**10.** What are all values of p for which the series $\sum_{n=1}^{\infty} \frac{1}{n^{2p}+n}$ diverges?

(A) $p \leq 1/2$

(B) p < 1/2 only

(C) $p \geq 1/2$

(D) p > 1/2 only

(E) The series diverges for all p.

**11.** For what values of $p$ will both series $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ and $\sum_{n=1}^{\infty} \left(\frac{p}{2}\right)^n$ converges?

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(A) $-2 < p < 2$ only

(B) $-\frac{1}{2} < p < \frac{1}{2}$ only

(C) $\frac{1}{2} < p < 2$ only

(D) $p < \frac{1}{2}$ and $p > 2$

(E) There are no such values of $p$.

**12.** What are all values of $p$ for which $\int_1^\infty \frac{1}{x^{2p}}dx$ converges?

(A) $p < -1$

(B) $p > 0$

(C) $p > \frac{1}{2}$

(D) $p > 1$

(E) There are no values of $p$ for which this integral converges.

**13.** Which of the following is a convergent $p$-series?

## Series

(A) $\sum_{n=1}^{\infty} n^3$

(B) $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$

(C) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(D) $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}}\right)^3$

**14.** Which of the following series converge?

$I. \sum_{n=1}^{\infty} \frac{1}{n^2}$ $II. \sum_{n=1}^{\infty} \frac{1}{n}$ $III. \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(A) I only

(B) III only

(C) I and II only

(D) I and III only

(E) I, II, and III

**15.** Which of the following series diverge?

I. $\sum_{k=3}^{\infty} \frac{2}{k^2+1}$

II. $\sum_{k=1}^{\infty} \left(\frac{6}{7}\right)^k$

III. $\sum_{k=2}^{\infty} \frac{(-1)^k}{k}$

(A) None

(B) II only

(C) III only

(D) I and III

(E) II and III

**16.** Which of the following series converges?

(A) $\sum_{n=1}^{\infty} \frac{3n}{n+2}$

(B) $\sum_{n=1}^{\infty} \frac{3n}{n^2+2}$

(C) $\sum_{n=1}^{\infty} \frac{3n}{n^2+2n}$

(D) $\sum_{n=1}^{\infty} \frac{3n^2}{n^3+2n}$

(E) $\sum_{n=1}^{\infty} \frac{3n^2}{n^4+2n}$

**17.** Which of the following series can be used with the limit comparison test to determine whether the series $\sum_{n=1}^{\infty} \frac{4^n}{5^n - n^2}$ converges or diverges?

## Series

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$

(B) $\sum_{n=1}^{\infty} \frac{1}{4^n}$

(C) $\sum_{n=1}^{\infty} \frac{1}{5^n}$

(D) $\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n$

**18.** Which of the following series can be used with the limit comparison test to determine whether the series $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ converges or diverges?

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$

(B) $\sum_{n=1}^{\infty} \frac{1}{n^3}$

(C) $\sum_{n=1}^{\infty} \frac{n}{n+1}$

(D) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

**19.** Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $\sum_{n=1}^{\infty} a_n$ converges, which of the following must be true?

## Series

(A) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(C) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(D) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $b_n \leq a_n$, then the behavior of $\sum_{n=1}^{\infty} b_n$ cannot be determined from the information given.

**20.** If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \leq a_n \leq b_n$ for all $n$, which of the following statements must be true?

(A) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

(B) $\sum_{n=1}^{\infty} (-1)^n b_n$ converges.

(C) $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges.

(D) $\sum_{n=1}^{\infty} b_n$ converges.

(E) $\sum_{n=1}^{\infty} b_n$ diverges.

Let $a_n = \frac{1}{n \ln n}$ for $n \geq 3$.

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**21.** Consider the infinite series $\sum_{n=3}^{\infty} (-1)^{n+1} a_n = \frac{1}{3 \ln 3} - \frac{1}{4 \ln 4} + \frac{1}{5 \ln 5} - \cdots$. Identify the properties of this series that guarantee the series coverage. Explain why the sum of this series is less than $\frac{1}{3}$.

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Please respond on separate paper, following directions from your teacher.

**22.** The power series $\sum_{n=1}^{\infty} \frac{(x-5)^n}{2^n n^2}$ has radius of convergence 2. At which of the following values of *x* can the alternating series test be used with this series to verify convergence at *x* ?

(A) 6

(B) 4

(C) 2

(D) 0

(E) -1

**23.** Which of the following statements are true about the series $\sum_{n=2}^{\infty} a_n$, where $a_n = \frac{(-1)^n}{\sqrt{n}+(-1)^n}$?

I. The series is alternating.
II. $|a_{n+1}| \leq |a_n|$ for all $n \geq 2$
III. $\lim_{n\to\infty} a_n = 0$

## Series

(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III

**24.** The alternating series test can be used to show convergence for which of the following series?

1. $1-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}+\frac{1}{25}-\frac{1}{36}+\cdots+a_n+\cdots$, where $a_n=(-1)^{n+1}\frac{1}{n^2}$

2. $\sin 1-\frac{\sin 2}{4}+\frac{\sin 3}{9}-\frac{\sin 4}{16}+\frac{\sin 5}{25}-\frac{\sin 6}{36}+\cdots+b_n+\cdots$, where $b_n=(-1)^{n+1}\frac{\sin n}{n^2}$

3. $\frac{1}{\sqrt{2}+1}-\frac{1}{\sqrt{2}-1}+\frac{1}{\sqrt{3}+1}-\frac{1}{\sqrt{3}-1}+\frac{1}{\sqrt{4}+1}-\frac{1}{\sqrt{4}-1}+\cdots+c_n+\cdots$,

where $c_n=\begin{cases}\frac{1}{\sqrt{k+1}+1} & \text{if } n=2k-1\\ -\frac{1}{\sqrt{k+1}-1} & \text{if } n=2k\end{cases}$

(A) I only

(B) II only

(C) I and II only

(D) I and III only

**25.** Which of the following series converges for all real numbers $x$ ?

(A) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(B) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(C) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

(D) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$

(E) $\sum_{n=1}^{\infty} \frac{n! x^n}{e^n}$

**26.** Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$

II. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$

III. $\sum_{n=1}^{\infty} \frac{n+1}{(n)(n+2)(n+3)}$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) I, II, and III

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**27.** What are all values of $x$ for which the series $\sum_{n=1}^{\infty} \frac{n3^n}{x^n}$ converges?

(A) All $x$ except $x = 0$

(B) $|x| = 3$

(C) $-3 \le x \le 3$

(D) $|x| > 3$

(E) The series diverges for all $x$.

**28.** What are all positive values of $p$ for which the series $\sum_{n=1}^{\infty} \frac{n^p}{4^n}$ will converge?

(A) $p > 0$

(B) $0 < p < 4$ only

(C) $p > 1$ only

(D) There are no positive values of $p$ for which the series will converge.

**29.** Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

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(A) $\lim_{n\to\infty} \frac{e}{n!} < 1$

(B) $\lim_{n\to\infty} \frac{n!}{e} < 1$

(C) $\lim_{n\to\infty} \frac{n+1}{e} < 1$

(D) $\lim_{n\to\infty} \frac{e}{n+1} < 1$

(E) $\lim_{n\to\infty} \frac{e}{(n+1)!} < 1$

**30.** Which of the following series are conditionally convergent?

$i.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

$ii.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

$iii.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

**31.** For what values of $p$ is the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^p + 2}$ conditionally convergent?

## Series

(A) $0 < p \leq 1$

(B) $p > 1$

(C) $1 < p \leq 2$ only

(D) $p > 2$ only

Let $f$ be the function given by $f(x) = e^{-2x^2}$.

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**32.** Let $g$ be the function given by the sum of the first four nonzero terms of the power series for $f(x)$ about $x=0$ . Show that $|f(x)-g(x)| <0.02$ for $-0.6 \leq x \leq 0.6$ .

Please respond on separate paper, following directions from your teacher.

**33.** The Taylor series for a function $f$ about $x = 0$ is given by $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!} x^{2n}$ and converges to $f$ for all real numbers $x$. If the fourth-degree Taylor polynomial for $f$ about $x = 0$ is used to approximate $f\left(\frac{1}{2}\right)$ alternating series error bound?

(A) $\frac{1}{2^4 \cdot 5!}$

(B) $\frac{1}{2^5 \cdot 6!}$

(C) $\frac{1}{2^6 \cdot 7!}$

(D) $\frac{1}{2^{10} \cdot 11!}$

## Series

The function $f$ is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$$

for all real numbers $x$.

**34.** Show that $1 - \frac{1}{3!}$ approximates $f(1)$ with error less than $\frac{1}{100}$.

Please respond on separate paper, following directions from your teacher.

**35.** If the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$ is approximated by the partial sum with 15 terms, what is the alternating series error bound?

(A) $\frac{1}{15}$

(B) $\frac{1}{16}$

(C) $\frac{1}{31}$

(D) $\frac{1}{33}$