Series

Name

The function f satisfies the equation $f'(x) = f(x) + x + 1$ and $f(0) = 2$. The Taylor series for f about $x = 0$ converges to $f(x)$ for all x.

1. Find $f''(0)$ and find the second-degree Taylor polynomial for f about $x = 0$.

Please respond on separate paper, following directions from your teacher.

2. Find the fourth-degree Taylor polynomial for f about $x = 0$.

Please respond on separate paper, following directions from your teacher.

3. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

x	0	$0<x<2$	2	$2<x<3$	3	$3<x<4$	4	$4<x<6$	6
$g(x)$	1	Positive	4	Positive	6	Positive	8	Positive	4
$g'(x)$	3	Positive	0	Positive	4	Positive	0	Negative	-4
$g''(x)$	-4	Negative	0	Positive	0	Negative	-2	Negative	-2

Let g be a twice-differentiable function. The function g and its derivatives have the properties indicated in the table above.

(a) Find the average rate of change of g' over the interval $[0, 6]$. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

(b) Find the x-coordinate of each point of inflection of the graph of g on the open interval $0 < x < 6$. Justify your answers.

Please respond on separate paper, following directions from your teacher.

(c) Use a right Riemann sum with the three subintervals $[0, 2]$, $[2, 3]$, and $[3, 4]$ to approximate $\int_0^4 g(x)\,dx$. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

(d) Is the approximation from part (c) an overestimate or underestimate for $\int_0^4 g(x)\,dx$? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

(e) Find the value of $\int_0^3 x \cdot g''(x)\,dx$. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

(f) Write the second-degree Taylor polynomial for g about $x = 4$.

Please respond on separate paper, following directions from your teacher.

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4. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

x	4	4.2	4.4
$g'(x)$	32	50	72

Let $y = g(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = g'(x)$ with initial condition $g(4) = 3$. Selected values of g' are given in the table above. The function g is three-times differentiable.

(a) Write an equation for the line tangent to the graph of g at $x = 4$.

Please respond on separate paper, following directions from your teacher.

(b) Let h be the function defined by $h(x) = 15 + x \cdot g(x)$. Find $h'(4)$.

Please respond on separate paper, following directions from your teacher.

(c) Use a right Riemann sum with the two subintervals indicated by the data in the table to approximate $\int_4^{4.4} g'(x)\,dx$. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

(d) It is known that $g''(x) > 0$ for $4 \leq x \leq 4.4$. Is the approximation from part (c) an overestimate or an underestimate for $\int_4^{4.4} g'(x)\,dx$? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

(e) Use the approximation for $\int_4^{4.4} g'(x)\,dx$ from part (c) to estimate the value of $g(4.4)$. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

(f) Use Euler's method, starting at $x = 4$ with two steps of equal size, to approximate $g(4.4)$. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

(g) It is known that $g''(4) = 80$ and $g'''(4) = 100$. Write the third-degree Taylor polynomial for g about $x = 4$.

Please respond on separate paper, following directions from your teacher.

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

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5. Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

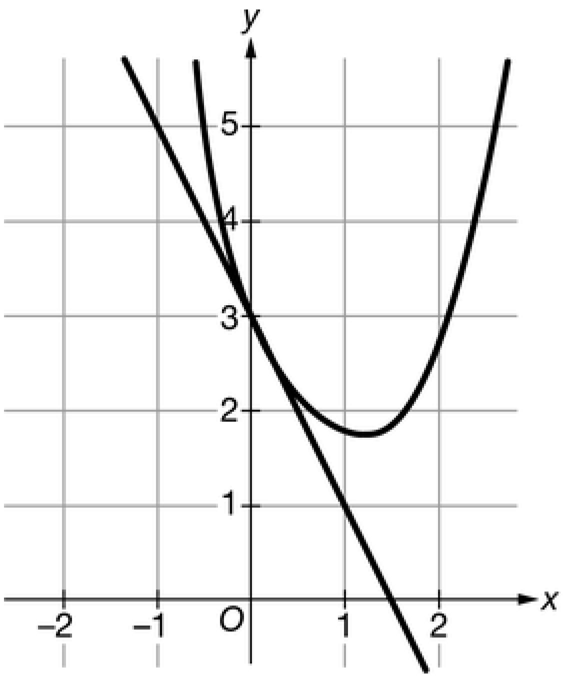
Please respond on separate paper, following directions from your teacher.

6. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



n	$f^{(n)}(0)$
2	3
3	$-\frac{23}{2}$
4	54

A function f has derivatives of all orders for all real numbers x. A portion of the graph of f is shown above, along with the line tangent to the graph of f at $x = 0$. Selected derivatives of f at $x = 0$ are given in the table above.

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a) Write the third-degree Taylor polynomial for f about $x = 0$.

Please respond on separate paper, following directions from your teacher.

b) Write the first three nonzero terms of the Maclaurin series for e^x. Write the second-degree Taylor polynomial for $e^x f(x)$ about $x = 0$.

Please respond on separate paper, following directions from your teacher.

c) Let h be the function defined by $h(x) = \int_0^x f(t)\,dt$. Use the Taylor polynomial found in part (a) to find an approximation for $h(1)$.

Please respond on separate paper, following directions from your teacher.

d) It is known that the Maclaurin series for h converges to $h(x)$ for all real numbers x. It is also known that the individual terms of the series for $h(1)$ alternate in sign and decrease in absolute value to 0. Use the alternating series error bound to show that the approximation found in part (c) differs from $h(1)$ by at most 0.45.

Please respond on separate paper, following directions from your teacher.

Let $P(x)=7-3(x-4)+5(x-4)^2-2(x-4)^3+6(x-4)^4$ be the fourth-degree Taylor polynomial for the function f about 4. Assume f has derivatives of all orders for all real numbers.

7. Find $f(4)$ and $f'''(4)$.

Please respond on separate paper, following directions from your teacher.

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8. Write the second-degree Taylor polynomial for f' about 4 and use it to approximate $f'(4.3)$.

Please respond on separate paper, following directions from your teacher.

Let f be the function given by $f(x) = e^{-2x^2}$.

9. Find the interval of convergence of the power series for $f(x)$ about $x = 0$. Show the analysis that leads to your conclusion.

Please respond on separate paper, following directions from your teacher.

Let f be the function given by <img src="/tmp/formula_5ff65bb0c60258.86971082_1609980848.svg" style="vertical-align:middle"> and G be the function given by <img src="/tmp/formula_5ff65bb0ce1a12.04434427_1609980848.svg" style="vertical-align:middle">.

10. Find the first four nonzero terms and the general term for the power series expansion of $G(x)$ about $x=0$.

Please respond on separate paper, following directions from your teacher.

11. Find the interval of convergence of the power series in part (b). (Your solution must include an analysis that justifies your answer.)

Please respond on separate paper, following directions from your teacher.

12. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$ converges?

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(A) $-3 \leq x \leq 3$

(B) -3<x<3

(C) $-1 < x \leq 5$

(D) $-1 \leq x \leq 5$

(E) $-1 \leq x < 5$

The function g has derivatives of all orders, and the Maclaurin series for g is $\sum_{n=0}^{\infty} (-1)^{n} \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots$.

13. Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

Please respond on separate paper, following directions from your teacher.

14. What is the radius of convergence for the power series $\sum_{n=0}^{\infty} \frac{(x-4)^n}{2 \cdot 3^{n+1}}$?

(A) $\frac{1}{3}$

(B) $\frac{3}{2}$

(C) 3

(D) 4

(E) 6

15. Let f be a function with second derivative $f''(x) = \sqrt{1+3x}$. The coefficient of x^3 in the Taylor series for f about $x = 0$ is

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

(E) $\frac{3}{2}$

16. What is the coefficient of x^6 in the Taylor series for $\frac{e^{3x^2}}{2}$ about $x = 0$?

(A) $\frac{1}{1440}$

(B) $\frac{81}{160}$

(C) $\frac{9}{4}$

(D) $\frac{9}{2}$

(E) $\frac{27}{2}$

17. If $f(x) = x \sin(2x)$, which of the following is the Taylor series for f aboutx=0 ?

(A) $x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \cdots$

(B) $x - \frac{4x^3}{2!} + \frac{16x^5}{4!} - \frac{64x^7}{6!} + \cdots$

(C) $2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!} + \cdots$

(D) $2x^2 - \frac{2x^4}{3!} + \frac{2x^6}{5!} - \frac{2x^8}{7!} + \cdots$

(E) $2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128x^8}{7!} + \cdots$

18. Which of the following is the Maclaurin series for $\frac{1}{(1-x)^2}$?

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(A) $1 - x + x^2 - x^3 + \ldots$

(B) $1 - 2x + 3x^2 - 4x^3 + \ldots$

(C) $1 + 2x + 3x^2 + 4x^3 + \ldots$

(D) $1 + x^2 + x^4 + x^6 + \ldots$

(E) $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots$

19. A function f has Maclaurin series given by $\frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \cdots + \frac{x^{n+3}}{(n+1)!} + \cdots$. Which of the following is an expression for $f(x)$?

(A) $-3x \sin x + 3x^2$

(B) $-\cos\left(x^2\right) + 1$

(C) $-x^2 \cos x + x^2$

(D) $x^2 e^x - x^3 - x^2$

(E) $e^{x^2} - x^2 - 1$

20. The sum of the series $1 + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \ldots + \frac{2^n}{n!} + \ldots$ is

(A) ln 2

(B) e^2

(C) cos 2

(D) sin 2

(E) nonexistent

21. What is the sum of the series $1 + \ln 2 + \frac{(\ln 2)^2}{2!} + \cdots + \frac{(\ln 2)^n}{n!} + \cdots$?

(A) $\ln 2$

(B) $\ln(1 + \ln 2)$

(C) 2

(D) e^2

(E) The series diverges.

The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$$

for all real numbers x.

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22. Show that $y = f(x)$ is a solution to the differential equation $xy' + y = \cos x$.

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Please respond on separate paper, following directions from your teacher.

23. For $x > 0$, the power series $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \ldots + (-1)^n \frac{x^{2n}}{(2n+1)!} + \ldots$ converges to which of the following?

(A) $\cos x$

(B) $\sin x$

(C) $\frac{\sin x}{x}$

(D) $e^x - e^{x^2}$

(E) $1 + e^x - e^{x^2}$

24. Which of the following is the Maclaurin series for e^{3x}?

(A) $1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

(B) $3 + 9x + \frac{27x^2}{2} + \frac{81x^3}{3!} + \frac{243x^4}{4!} + \cdots$

(C) $1 - 3x + \frac{9x^2}{2} - \frac{27x^3}{3!} + \frac{81x^4}{4!} - \cdots$

(D) $1 + 3x + \frac{3x^2}{2} + \frac{3x^3}{3!} + \frac{3x^4}{4!} + \cdots$

(E) $1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{3!} + \frac{81x^4}{4!} + \cdots$

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25. The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{x^2}{1-x^2}$?

(A) $1+x^2+x^4+x^6+x^8+\cdots$

(B) $x^2+x^3+x^4+x^5+\cdots$

(C) $x^2+2x^3+3x^4+4x^5+\cdots$

(D) $x^2+x^4+x^6+x^8+\cdots$

(E) $x^2-x^4+x^6-x^8+\cdots$

26. What is the coefficient of x^2 in the Taylor series for $\frac{1}{(1+x)^2}$ about $x = 0$?

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) 1

(D) 3

(E) 6

27. The Maclaurin series for the function f is given by $f(x) = \sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$. What is the value of $f(3)$?

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(A) -3

(B) $-\frac{3}{7}$

(C) $\frac{4}{7}$

(D) $\frac{13}{16}$

(E) 4

Let f be a function that has derivatives of all orders for all real numbers. Assume $f(1)=3, f'(1) =-2, f''(1)= 2$ and $f'''(1)=4$

28. Write the second-degree Taylor polynomial for f', the derivative of f, aboutx=1 and use it to approximate $f'(1.2)$.

Please respond on separate paper, following directions from your teacher.

Let f be the function given by <img src="/tmp/formula_5ff65bb41cca35.49546199_1609980852.svg" style="vertical-align:middle">.

29. Write the first four nonzero terms and the general term for the Taylor series expansion of $f(x)$ about $x=0$.

Please respond on separate paper, following directions from your teacher.