10. Why can you swap a horizontal transformation and if a ertical transformation without changing the overall transformation?

$H: (x, y) \mapsto (bx + c, y)$

$V: (x, y) \mapsto (x, ay + d)$

$H \circ V: (x, y) \xmapsto{V} (x, ay + d) \xmapsto{H} (bx + c, ay + d)$

$V \circ H: (x, y) \xmapsto{H} (bx + c, y) \xmapsto{V} (bx + c, ay + d)$

A horizontal translation does not affect the vertical ones, vice versa.

11. Apply the **horizontal** transformations $T_3 = A \circ B \circ C \circ D \circ E \circ B \circ C$ put it in mapping and function form and simplify to a standard stretch and then shift form.

$T_3: (x, y) \xmapsto{C} (-x, y) \xmapsto{B} (-x - 5, y) \xmapsto{E} \left(\frac{-x-5}{2}, y\right) \xmapsto{D} \left(\frac{-x-5}{2} + 1, y\right) \xmapsto{C} \left(\frac{x+5}{2} + 1, y\right)$

$\xmapsto{B} \left(\frac{x+5}{2} - 1 - 5, y\right) = \left(\frac{x+5}{2} - 6, y\right) \xmapsto{A} \left(\frac{3}{2}(x+5) - 18, y\right)$

$T_3: (x, y) \mapsto \left(\frac{3}{2}x - \frac{21}{2}, y\right) \Rightarrow$ expand horiz. by $\frac{3}{2}$ then left by $\frac{21}{2}$

$g: \underbrace{\frac{3}{2}x - \frac{21}{2}}_{X} \mapsto y = f(x) = g(X) = f\left(\frac{2X + 21}{3}\right) = f\left(\frac{2}{3}X + 7\right)$

$X = \frac{3}{2}x - \frac{21}{2}$

$x = \frac{2X + 21}{2}$

When we invert the x, y values get swapped with the transformation:

$$I: (x, y) \mapsto (y, x)$$

And consider the transformation:

$$T: (x, y) \mapsto (x + 3, 2y)$$

After this transformation we get that

$$g(x) = 2f(x - 3)$$

12. Apply the transformation $T_4 = T \circ I$ and reason why

$$h(x) = 2f^{-1}(x - 3)$$

$T_4: (x, y) \xmapsto{I} (y, x) \xmapsto{T} (y+3, 2x)$ ⇒ We have transformed f^{-1} by right 3 and vertically expand by 2.

$f: x \mapsto y = f(x) \Rightarrow x = \boxed{f^{-1}(y)}$

$h: y+3 \mapsto 2x = h(y+3) = 2\boxed{f^{-1}(y)}$

Let $Y = y+3 \Rightarrow h(Y) = 2f^{-1}(Y-3)$

$y = Y - 3$ $\qquad h(x) = 2f^{-1}(x-3)$

Just a dummy variable

We can change Y back to x because h is just transforming Y the same way as how we normally transform x into h(x) (*Because we have already inverse it in the first step)

13. Apply the transformation $T_5 = I \circ T$ and reason why

$$k(x) = f^{-1}\left(\frac{1}{2}x\right) + 3$$

METHOD 1:

$T_5: (x, y) \xmapsto{T} (x+3, 2y) \xmapsto{I} (2y, x+3)$

$f: x \mapsto y = f(x) \Rightarrow x = f^{-1}(y)$

$K: 2y \mapsto x+3 = K(2y)$

$f^{-1}(y) + 3 = K(2y)$

let $Y = 2y \Rightarrow K(Y) = f^{-1}\left(\frac{Y}{2}\right) + 3$

$\frac{Y}{2} = y$ $\qquad K(x) = f^{-1}\left(\frac{x}{2}\right) + 3$

METHOD 2: Direct Function Notation

Transform: $g(x) = 2f(x-3)$

Inverse: $x = 2f(y-3)$

$\frac{1}{2}x = f(y-3)$

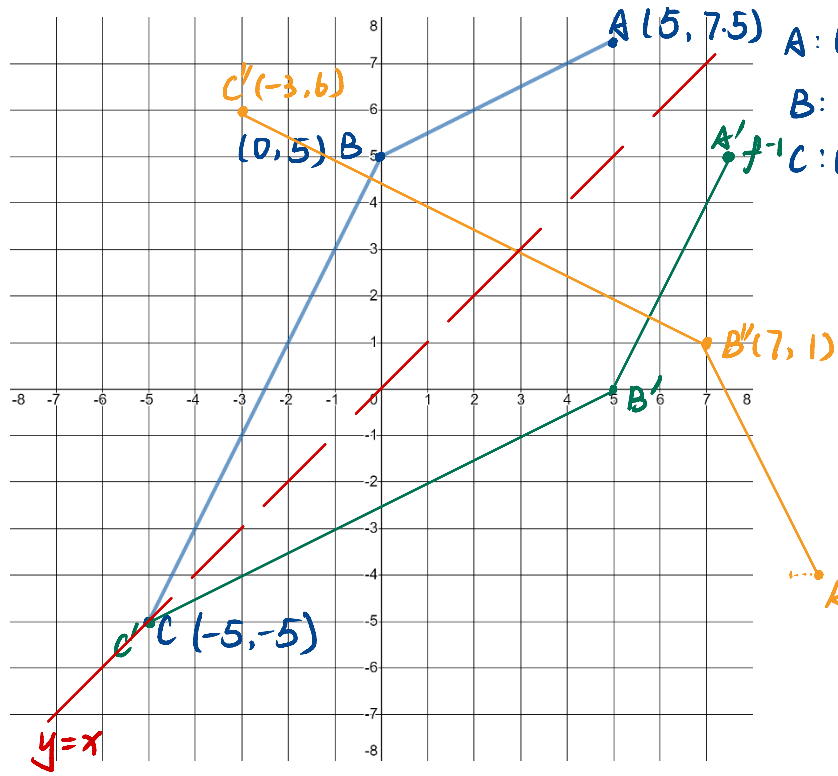
$f^{-1}\left(\frac{1}{2}x\right) = \cancel{f^{-1}}\cancel{f}(y-3)$

$k(x) = g^{-1}(x) = y = f^{-1}\left(\frac{1}{2}x\right) + 3$

Given the following graph of f, and the transformation:

$$T:(x,y)\mapsto(x+2,-y+1) \text{ and } I:(x,y)\mapsto(y,x)$$

14. Graph the transformation of f under $T\circ I$ and write the function notation of the transformed function



$A:(5,7.5)\xmapsto{I}(7.5,5)\xmapsto{T}(9.5,-4)$

$B:(0,5)\xmapsto{I}(5,0)\xmapsto{T}(7,1)$

$C:(-5,-5)\xmapsto{I}(-5,-5)\xmapsto{T}(-3,6)$

Function Notation:

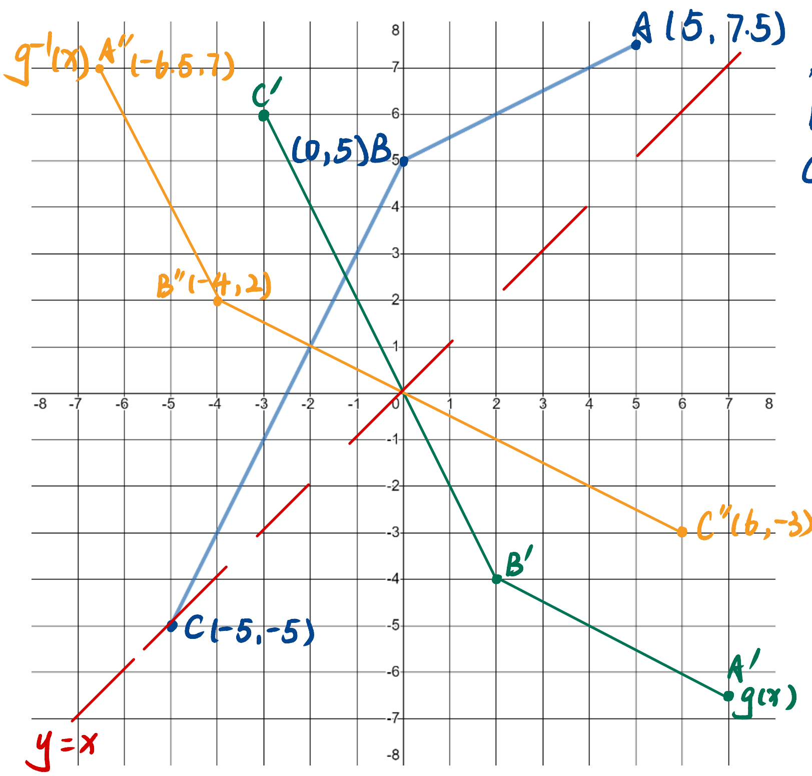
$I: y=f^{-1}(x)$

T: ① Right 2: $f^{-1}(x-2)$

② Reflect over y axis: $-f^{-1}(x-2)$

③ Up 1 unit: $\boxed{-f^{-1}(x-2)+1}$

15. Graph the transformation of f under $I\circ T$ and write the function notation of the transformed function



$A:(5,7.5)\xmapsto{T}(7,-6.5)\xmapsto{I}(-6.5,7)$

$B:(0,5)\xmapsto{T}(2,-4)\xmapsto{I}(-4,2)$

$C:(-5,-5)\xmapsto{T}(-3,6)\xmapsto{I}(6,-3)$

Function Notation:

T: ① Right 2: $f(x-2)$

② Reflect over y-axis: $-f(x-2)$

③ Up 1 unit: $-f(x-2)+1$

$I: x=-f(y-2)+1$

$-x+1=-f(y-2)$

$f^{-1}(-x+1)=\cancel{f^{-1}}f(y-2)$

$y=f^{-1}(-x+1)+2$

$\boxed{y=f^{-1}(-(x-1))+2}$