Trig Models: Understanding Practice 5

Goal: Be able to model trig functions in general cases and interpret the model in a meaningful way.

Given the following situations, what are important numbers (constants and variables) that are needed to understand and predict things, and what would we want to predict about the situation? **Identify the period.**

A. Sales of winter jackets are seasonal.

B. Breathing in and out is a regular bodily function.

C. A wind turbine has three blades that spin.

D. The phase of the moon is regular.

1. In situation A, I imagine a sales of winter jackets at MEC are seasonal with the most occurring in November at 400 units sold and basically none being sold 6 months later.

   Determine a function for the number of jackets sold in terms of the month.

$$J(m) = 200\cos\left(\frac{2\pi}{12}(m-11)\right) + 200$$

$m=0$ is Dec $\quad m=1$ is Jan

2. Generalize this. The number of coats sold in November is N and the number sold 6 months later is 0. State the mapping notation of this function and describe its domain.

$$J(m,N) = \frac{N}{2}\cos\left(\frac{2\pi}{12}(m-11)\right) + \frac{N}{2}$$

$$J : \mathbb{Z} \times \mathbb{N} \cup \{0\} \to \mathbb{N} \cap [0, N]$$

$m \in \mathbb{Z}$ since $m=-1$ is November of last year

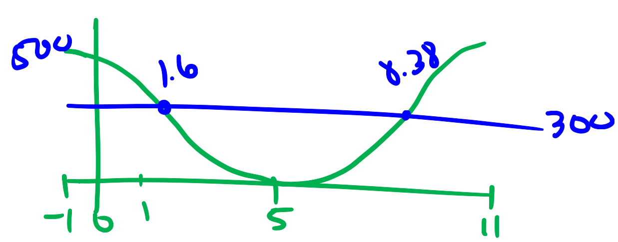
3. Use the situation if $N = 500$ and determine the interval that they sell 300 jackets.

$$J(m, 500) = 250\cos\left(\frac{\pi}{6}(m-11)\right) + 250 = 300$$

$$\Rightarrow \cos\theta = \frac{50}{250} \Rightarrow \theta = \pm 1.37 + 2\pi n = \frac{\pi}{6}(m-11)$$

$$\Rightarrow \pm 2.62 + 12n = m - 11$$

$$\Rightarrow 8.38 \text{ or } \underset{=1.6}{13.6} + 12n$$



They sell over 300 Jackets per month during September through December

4. In situation B I imagined a normal adult breathes in and exhales 0.84 L of air every 4 seconds. When they exhale, the minimal amount in the lungs of 0.08 L.

   Determine a function for the volume of air in their lungs at time t.

$$V(t) = 0.42\cos\left(\frac{2\pi}{4}(t)\right) + 0.5$$

just pick cosine cause its easier to measure a breath starting at a full breath

5. Generalize this. A person breathes in and out once every t_0 seconds. They inhale and exhale V litres of air and a minimum capacity of v in litres. State the mapping notation of this function and describe its domain.

$$\mathbb{V}(t_0, V, v, t) = \frac{V}{2}\cos\left(\frac{2\pi}{t_0}t\right) + v + \frac{V}{2}$$

$$\mathbb{V} : \mathbb{Q}^4 \to \mathbb{Q} \cap [v, v+V]$$

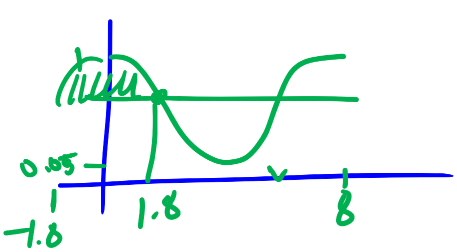
$t_0, V, v > 0$ as you need time to take a breath and your lungs can't collapse

6. Use the situation where $t_0 = 8$, $V = 0.95$ and $v = 0.05$ to determine how long in one cycle they will have more than 0.6 L of air in their lungs.

$$\mathbb{V}(8, 0.95, 0.05, t) = 0.475\cos\left(\frac{\pi}{4}t\right) + 0.525 = 0.6$$

$$\Rightarrow \cos\left(\frac{\pi}{4}t\right) = 0.16 \quad \Rightarrow \frac{\pi}{4}t = \pm 1.41 + 2\pi n$$

$$\Rightarrow t = \pm 1.80 + 8n$$



in one cycle they have > 0.6 L of air for 3.6 sec (out of 8 sec)

or 45% of the time

7. In situation C I imagined a turbine that has three blades that are 120 feet long and the propeller spins once every 5 seconds. The center of the turbine is 320 feet above the ground and the blades are spaced evenly apart.

amp

period

midline

Determine a function for the height of each blade above the ground at time t.

$$h_1(t) = 120\cos\left(\frac{2\pi}{5}t\right) + 320$$

$$h_2(t) = h_1\left(t + \frac{5}{3}\right)$$

go back $\frac{1}{3}$ of a rotation

$$h_3(t) = h_1\left(t - 5/3\right)$$

go forward $\frac{1}{3}$ of a rotation

8. Generalize this. Each blade is b feet long, the turbine is H feet tall, and the propeller spins once every t_0 seconds. State the mapping notation of this function and describe its domain.

$$h_1(b, H, t_0, t) = b\cos\left(\frac{2\pi}{t_0}t\right) + H$$

$$h_1 : \mathbb{N}^2 \times \mathbb{Q}^2 \to \mathbb{Q} \cap [H-b, H+b]$$

$b, H \in \mathbb{N}$ as they are likely so large they are just reported to the nearest foot.

$t_0, t \in \mathbb{Q}$ and $t_0 > 0$ since it take time to rotate.

$$h_2 = h_1\left(b, H, t_0, t - \frac{t_0}{3}\right) \quad h_3 = h_1\left(b, H, t_0, t + \frac{t_0}{3}\right)$$

9. In situation D I imagined that the phase of the moon follows a sinusoidal path. On February 11, 2021 there was a new moon (it was 0% visible) and on February 27, 2021 there was a full moon (100% visible).

   Make a function for the percent the moon is visible as a function of the days in **March**.

$$M(d) = 0.5\cos\left(\frac{\pi}{16}(d+1)\right) + 0.5$$

Feb 27 − Feb 11 = 16 days ($\frac{1}{2}$ period)

$t=1$ is March 1 $t=-1=$ Feb 27 $t=-17=$ Feb 11

10. Generalize this. There is a full moon on day f in a month with m days and a new moon on day n (in the same month) and we want to know about the next month . State the mapping notation of this function and describe its domain.

$$M(f,n,m,d) = 0.5\cos\left(\frac{\pi}{f-n}\left(d-(f-m)\right)\right) + 0.5$$

☆ because cosine is even it doesn't matter if $f > n$

$M: \mathbb{N}^3 \times \mathbb{Z} \to [0,1] \cap \mathbb{Q}$ d can be negative as those are days in the past

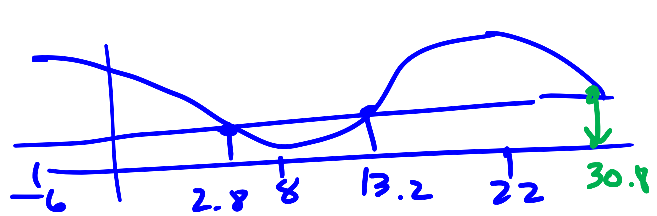
11. Use the situation where there will be a new moon on June 10, 2021 and a full moon on June 24. Use it to determine the days in **July** when the moon is more than 30% visible.

$$M(24,10,30,d) = 0.5\cos\left(\frac{\pi}{14}(d+6)\right) + 0.5 = 0.3$$

$$\cos\theta = -0.4 \Rightarrow \theta = \pm 1.98 + 2\pi n = \frac{\pi}{14}(d+6)$$

$$\Rightarrow d+6 = \pm 8.83 + 28n$$

$$\Rightarrow d = -14.8 \text{ or } 2.8 + 28$$
$$= 13.2$$



The moon will be more than 30% visible from July 1-2 and July 14-30