Exponential and Log Transformations
Understanding Practice 7

Goal: Use exponential and log laws to relate transformations into different forms of each other.

1. Consider the transformation of the exponential function:

$$f(x) = 2 \cdot e^{0.5(x-4)} - 1$$

   a. State how f has been transformed from e^x

   b. Write this transformation in the form $Ae^{kx} + D$, state the transformations that have occurred.

   c. Write this transformation in the form $e^{k(x-C)} + D$, state the transformations that have occurred.

2. Consider the transformation of the log function:

$$h(x) = 2\ln\left(\frac{1}{3}(x+3)\right) + 1$$

a. State how h has been transformed from $\ln x$.

b. Write this transformation in the form $A \cdot \ln(x - C) + D$, state the transformations that have occurred.

c. Write this transformation in the form $A \cdot \ln\big(B(x - c)\big)$, state the transformations that have occurred.

3. Consider the general transformation of the exponential function:

$$g(x) = a \cdot e^{k(x-c)} + d$$

a. Write g in the form $Ae^{Kx} + D$, where A, K, D are in terms of a, k, c, d. State which transformations are equivalent.

b. Write g in the form $e^{K(x-C)} + D$, where C, K, D are in terms of a, k, c, d. State which transformations are equivalent.

c. Why would the form in part (a) preferred over (b)? Hint: Consider what transformation couldn't occur if there were no vertical stretch.

d. Part (a) states a horizontal shift by X_0 is equivalent to a vertical stretch by $\mathcal{F}(X_0)$ where $\mathcal{F}$ is some relation. Part (b) says something similar, stating a vertical stretch by Y_0 is equivalent to a horizontal shift by $\mathcal{G}(Y_0)$ where $\mathcal{G}$ is some relation. Show that these equivalences are the same and show that $\mathcal{G} = \mathcal{F}^{-1}$.

4. Consider the general transformation of the log function:

$$k(x) = a \cdot \ln\big(b(x - c)\big) + d$$

   a. Write k in the form $A \cdot \ln(x - C) + D$, where A, C, D are in terms of a, b, c, d. State which transformations are equivalent.

   b. Write k in the form $A \cdot \ln\big(B(x - C)\big)$, where A, B, C are in terms of a, b, c, d. State which transformations are equivalent.

c. Why would the form in part (b) preferred over (a)?

d. Part (a) states a horizontal stretch by X_0 is equivalent to a vertical shift by $\mathcal{F}(X_0)$ where $\mathcal{F}$ is some relation. Part (b) says something similar, stating a vertical shift by Y_0 is equivalent to a horizontal stretch by $\mathcal{G}(Y_0)$ where $\mathcal{G}$ is some relation. Show that these equivalences are the same and show that $\mathcal{G} = \mathcal{F}^{-1}$.