Functions and Proper Notation

Goal:
- Can use function notation adeptly and understands how to read the language.
- Can model function behaviour.

Terminology:
- Composition
- One-to-one
- Inverse

Review: How can you represent the following relationships?

Domain Range

$m \mapsto 2(m+1)^3$

"A number is twice the cube of one more than some other number"

$f(m) = n = 2(m+1)^3$ OR $(2x)^3 + x$

$2x^3 +$

$2(1)^3 + x$

$\frac{\sqrt[3]{n}}{2} - 1 = m$ inverse

graph  $\longleftrightarrow$

m	n
0	2
1	16

$\longleftrightarrow \{(0,2), (1,16)\}$

"A number is four more than some other number divided by one less than the same number squared"

$y = 4 + \frac{x}{x^2-1} = g(x)$ function notation

$x \mapsto \frac{x}{x^2-1} + 4$

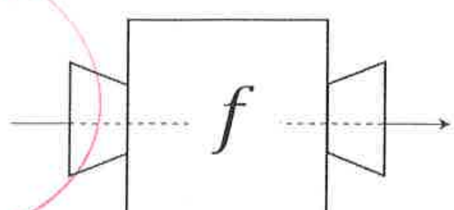
x	$g(x)$
0	4
1	? nothing

is not in domain

In both cases these are basic instructions that have a recipe like structure to get to the final result. First do this, then do this, and so one. I like to think of functions as little machines that take an input (a value from the domain) and spit out an output (a value that will be in the range).

$f(x) = 2(x+1)^3$

must be in domain of f

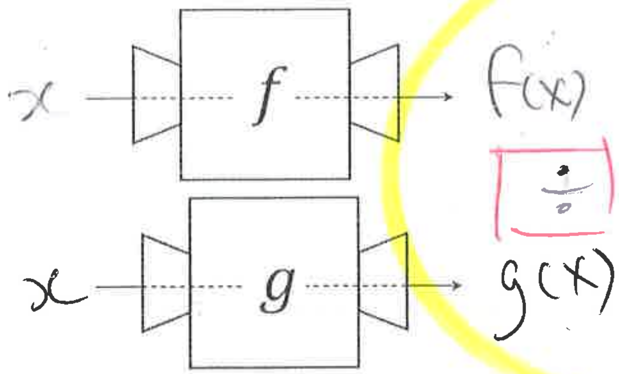


$f(♡) = (♡+1)^3 \cdot 2$

must be the same if we give f the same thing

1.) add 1
2.) cube it
3.) times by 2

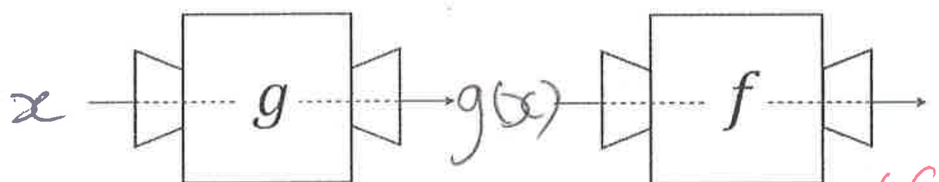
These machines can then be combined in any way you can imagine. Consider we have two functions f and g whose domain and range are both $\mathbb{R}$. Then we could run each function together and do some operation with the outputs,



combine using $+ / - / \times / \div$

Or we could run the functions in series with each other as a **composition**

f of g



$f(g(x))$

$(f \circ g)(x)$

Example: Given that $g(x) = x^2 + 3x$ and $f(x) = x \sin x$. Determine $f(f(1) - g(1))$

$f(1) - g(1) = 1\sin 1 - [(1)^2 + 3(1)]$

$= -3.158$ ☆ in radians or $-3.98\ldots$ ☆ in degrees

$f(-3.158) = -3.158 \sin(-3.158)$

$= -0.053$ ☆ radians

Practice: Determine $g\left(\frac{f(2)}{g(-2)}\right)$

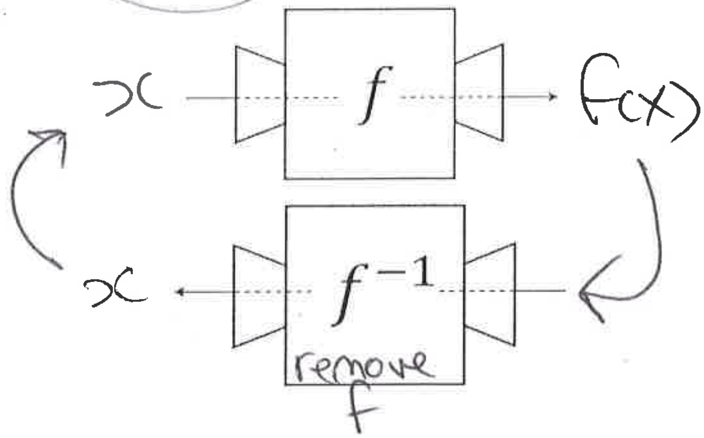
$\frac{f(2)}{g(-2)} = \frac{2\sin 2}{(-2)^2 + 3(-2)} = -0.909\ldots$

$g(-0.909) = (-0.909)^2 + 3(-0.909)$

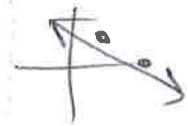
$\boxed{= -1.901\ldots}$

if it passes the horizontal line test

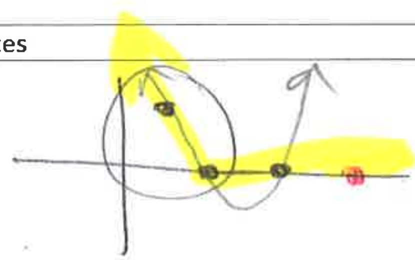
Also remember that if the function is **one-to-one,** we can send things backward through the function as the **inverse function** that undoes the function and we are finished with what we started with.



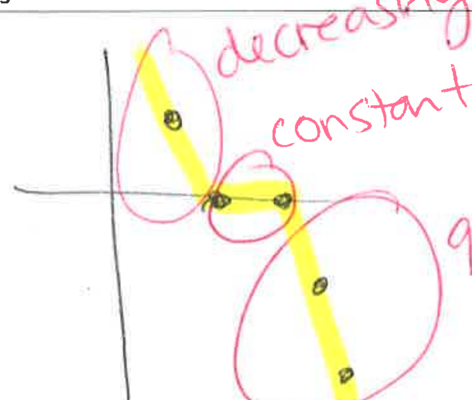
On the Board: Design a function that obeys, $f(1) = 1$ and $f(2) = 0$.

Solution	Notes
$f(x) = 2 - x$	what is $f(3) = ?$ maybe -1 

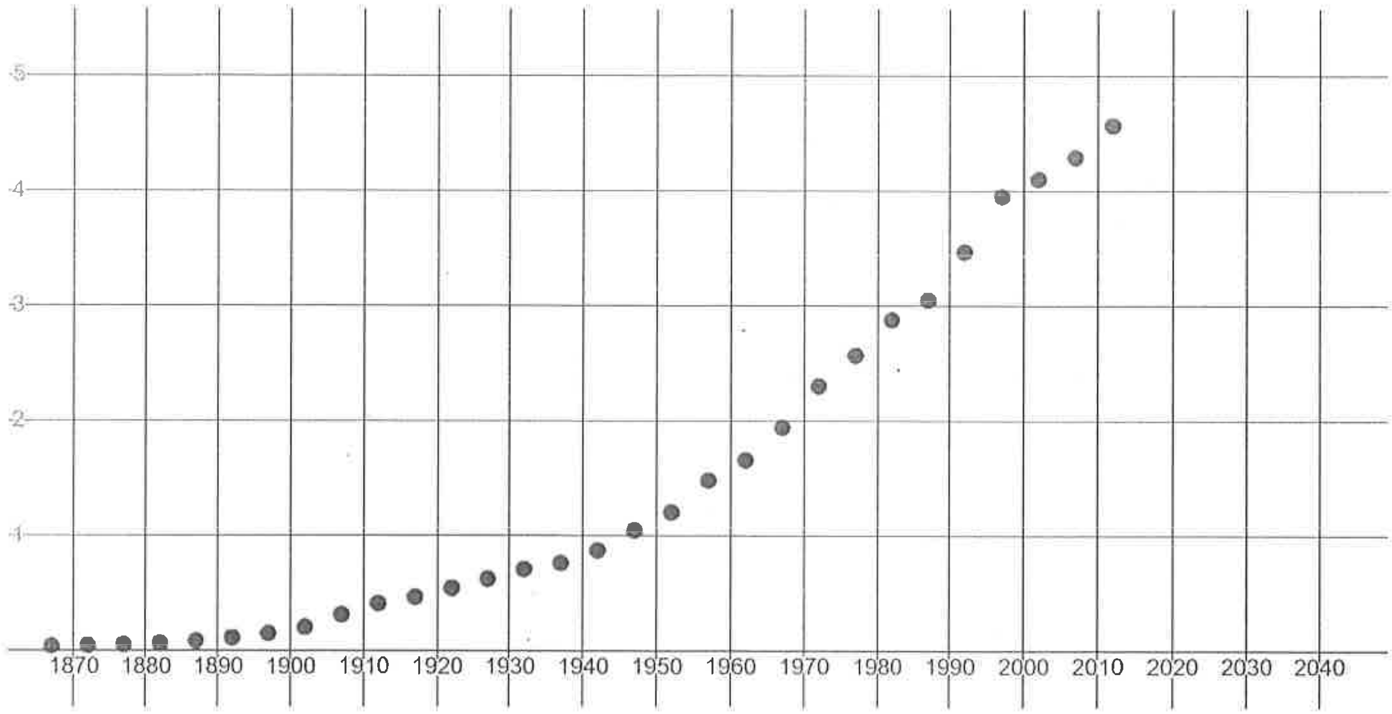
Adjust your above function so that $f(3) = 0$ too.

Solution	Notes
$f(x) = \begin{cases} 2-x, & x < 2 \\ 0, & x \geq 2 \end{cases}$	 $f(4) = ?$

One more adjustment so that $f(4) = -2$ and $f(5) = -4$.

Solution	Notes
	 decreasing constant quickly decreasing

On the Board: Design a function that mimics the following graph of the population of BC over the past 150 years (measured in millions). Your goal is to make the most accurate model possible for the domain $t \in [1870, 2040)$.



t	1867	1887	1897	1907	1917	1927	1937	1947	1957	1967	1977	1987	1997	2007	2017
$P(t)$	0.03	0.08	0.15	0.31	0.46	0.62	0.76	1.04	1.48	1.94	2.57	3.05	3.95	4.29	4.86

Source: https://www2.gov.bc.ca/gov/content/data/statistics/people-population-community/population/population-estimates

Practice Problems: Functions and Graphs Handout: # 31-34, 41-52, 54 (just find an equation for the data – don't worry about the words quadratic regression)

35, 36, 53

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Terminology: • Composition • One-to-one • Inverse	

Review: How can you represent the following relationships?

"A number is twice the cube of one more than some other number"

$y = 2(x+1)^3 = f(x)$ function notation

output #

x	$y = 2(x+1)^3$
0	2
4	250

Table $\iff \{(0,2), (4,250)\}$ ordered pair $\iff$ graph

Domain $x \mapsto 2(x+1)^3$ Range

Fundamental Relationship

"A number is four more than some other number divided by one less than the same number squared"

$g(x) = y = 4 + \frac{x}{(x^2-1)}$

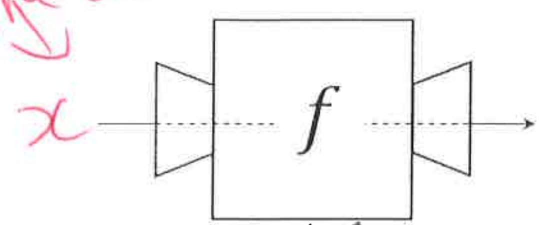
$0 \mapsto 4$

$\boxed{-1 \mapsto \text{undefined}}$ -1 is not in the domain

$x \mapsto 4 + \frac{x}{x^2-1}$

In both cases these are basic instructions that have a recipe like structure to get to the final result. First do this, then do this, and so one. I like to think of functions as little machines that take an input (a value from the domain) and spit out an output (a value that will be in the range).

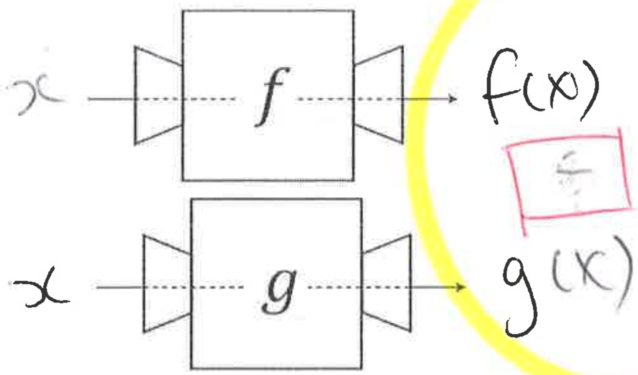
must be in the domain



must be unique (can't get different results with the same x)

1) add 1
2) cube it
3) double it

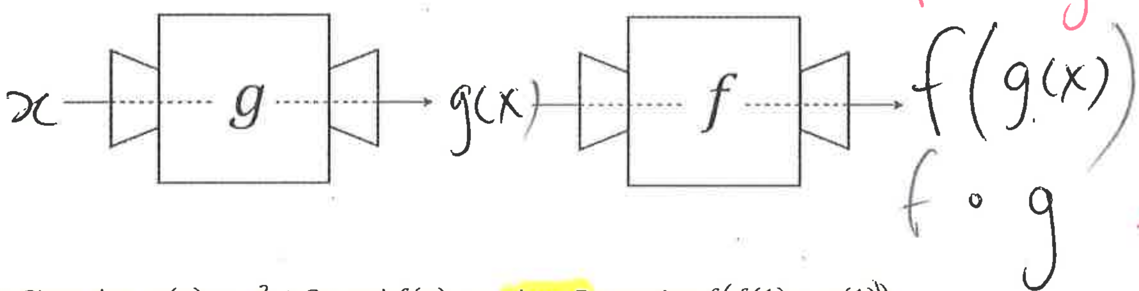
These machines can then be combined in any way you can imagine. Consider we have two functions f and g whose domain and range are both $\mathbb{R}$. Then we could run each function together and do some operation with the outputs,



combine using $+ / - / \times / \div$

Or we could run the functions in series with each other as a **composition**

f of g



Example: Given that $g(x) = x^2 + 3x$ and $f(x) = x \sin x$. Determine $f(f(1) - g(1))$

$f(1) - g(1) = 1\sin 1 - 1^2 - 3(1) = -3.98\ldots$

$f(-3.98) = -3.98 \sin(-3.98) = 0.277\ldots$ ☆ in degrees

Practice: Determine $g\left(\frac{f(2)}{g(-2)}\right)$

$\frac{f(2)}{g(-2)} = \frac{2\sin 2}{(-2)^2 + 3(-2)} = -0.91$ ☆ radians

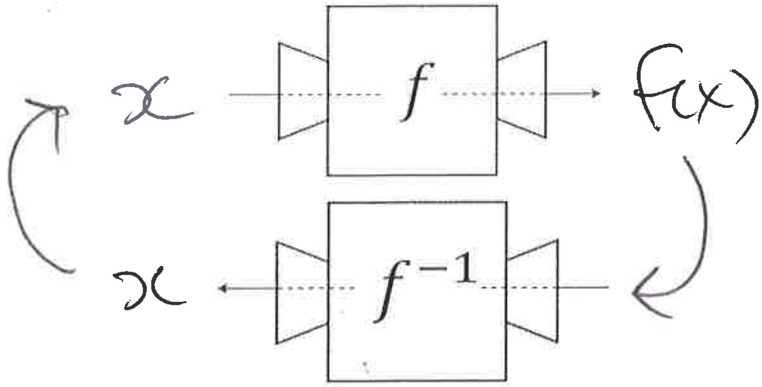
$= \cancel{1.81}\ -0.035$ ☆ in degrees

$g(-0.035) = (-0.035)^2 + 3(-0.035)$

$\boxed{= -0.103}$

pass the horizontal line test

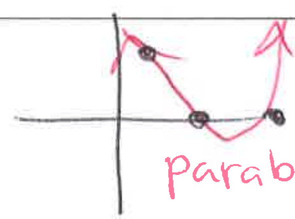
Also remember that if the function is one-to-one, we can send things backward through the function as the **inverse function** that undoes the function and we are finished with what we started with.



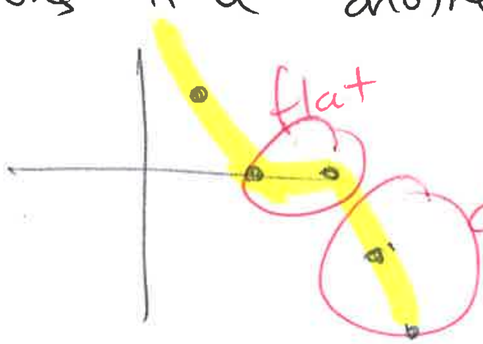
On the Board: Design a function that obeys $f(1) = 1$ and $f(2) = 0$.

Solution	Notes
$f(x) = 2 - x$	Two points connect with a straight line

Adjust your above function so that $f(3) = 0$ too.

Solution	Notes
$f(x) = \dfrac{(x-2)(x-3)}{2}$	looked like zeros $a(x-2)(x-3)$  parabola-esque?

One more adjustment so that $f(4) = -2$ and $f(5) = -4$.

Solution	Notes
$f(x) = \begin{cases} 2-x, & x<2 \\ 0, & 2 \le x \le 3 \\ -2x+6, & x>3 \end{cases}$	looks like another line  flat decreasing quickly

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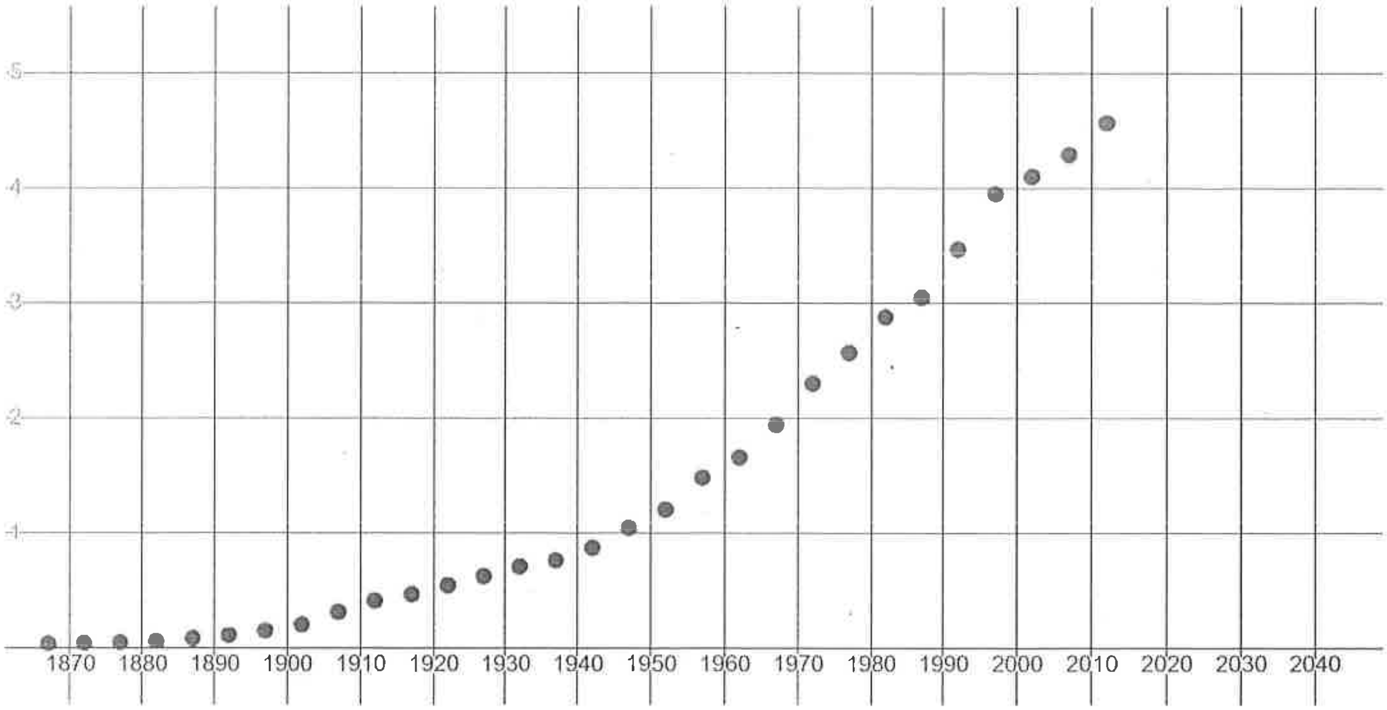

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