## Functions and Proper Notation

## Goal:

- Can use function notation adeptly and understands how to read the language.
- Can model function behaviour.

Terminology:

- Composition
- One-to-one
- Inverse

Review: How can you represent the following relationships?
"A number is twice the cube of one more than some other number"
"A number is four more than some other number divided by one less than the same number squared"

In both cases these are basic instructions that have a recipe like structure to get to the final result. First do this, then do this, and so one. I like to think of functions as little machines that take an input (a value from the domain) and spit out an output (a value that will be in the range).


These machines can then be combined in any way you can imagine. Consider we have two functions $f$ and $g$ whose domain and range are both $\mathbb{R}$. Then we could run each function together and do some operation with the outputs,


Or we could run the functions in series with each other as a composition


Example: Given that $g(x)=x^{2}+3 x$ and $f(x)=x \sin x$. Determine $f(f(1)-g(1))$

Practice: Determine $g\left(\frac{f(2)}{g(-2)}\right)$

Also remember that if the function is one-to-one, we can send things backward through the function as the inverse function that undoes the function and we are finished with what we started with.


On the Board: Design a function that obeys $f(1)=1$ and $f(2)=0$.

| Solution | Notes |
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Adjust your above function so that $f(3)=0$ too.

| Solution | Notes |
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One more adjustment so that $f(4)=-2$ and $f(5)=-4$.

| Solution | Notes |
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On the Board: Design a function that mimics the following graph of the population of BC over the past 150 years (measured in millions). Your goal is to make the most accurate model possible for the domain $t \in[1870,2040$ ).


| $t$ | 1867 | 1887 | 1897 | 1907 | 1917 | 1927 | 1937 | 1947 | 1957 | 1967 | 1977 | 1987 | 1997 | 2007 | 2017 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $P(t)$ | 0.03 | 0.08 | 0.15 | 0.31 | 0.46 | 0.62 | 0.76 | 1.04 | 1.48 | 1.94 | 2.57 | 3.05 | 3.95 | 4.29 | 4.86 |

Source: https://www2.gov.bc.ca/gov/content/data/statistics/people-population-community/population/populationestimates

Practice Problems: Functions and Graphs Handout: \# 31-34, 41-52, 54 (just find an equation for the data - don't worry about the words quadratic regression)

