Applied Rates of Change

Goal:

- Can use derivative rules to find appropriate rate of change for the context of the problem.
- Can use chain rule fluently to find rate of change with respect to time.

Terminology:

- Differential Equation

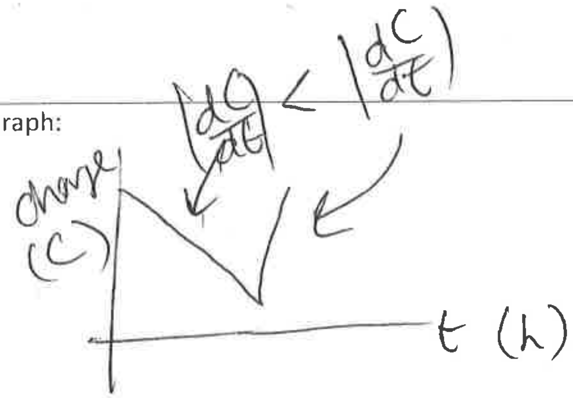
In your groups you need to make up 3 stories that relate one unit to another. I want to see a reasonable graph that describes the scenario and then an equation that could describe the graph. To help you choose your units we have a handy chart that will tell you what relation you should be looking at.

Birthday Month	Unit	Birthday Month	Unit	Birthday Month	Unit
January	Volume (m^3)	May	Cost ($)	September	Humidity (%)
February	Volume (L)	June	Memory (MB)	October	Population (people)
March	Temperature (°C)	July	Force (N)	November	Charge (C - Coulomb)
April	Pressure (atm)	August	Mass (kg)	December	Light Intensity (cd – candela)

Scenario #1 Person 1 vs. time

Description:

Graph:



Equation:

$$C(t) = \begin{cases} -5t + 100, & t < 12 \\ 20t + b, & t \geq 12 \end{cases}$$

Differential Equation:

$$\frac{dC}{dt} = \begin{cases} -5, & t < 12 \\ 20, & t \geq 12 \end{cases}$$

Scenario #1 Person 2 vs. Any unit that is not time

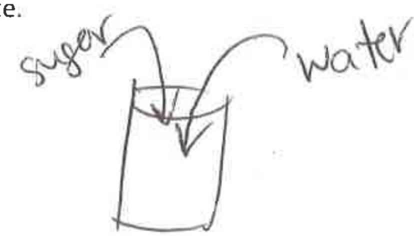
Description:

Graph:

Equation:	Differential Equations:

Example: Concentration is measure in amount of substance per unit volume. Imagine we start with a empty glass and add water at some variable rate and add sugar crystals at some other rate.

(a) Write an equation to model the concentration
(b) make a differential equation with respect to time
(c) make a differential equation with respect to amount of sugar



★ when I add sugar
let m be the mass of sugar (g) $\frac{dm}{dt}$ variable

whe I add water
let V be the volume of water (L)
$\frac{dV}{dt}$ also variable

★ units are related

(a) let C be concentration $C = \frac{m}{V}$

(b) $\frac{d}{dt}\left[C = \frac{m}{V}\right] \Rightarrow \frac{dC}{dt} = \frac{V\frac{dm}{dt} - m\frac{dV}{dt}}{V^2}$ ★ rates are related

Variables: V, m, $\frac{dm}{dt}$, $\frac{dV}{dt}$, $\frac{dC}{dt}$

(c) $\frac{d}{dm}\left[C = \frac{m}{V}\right] \Rightarrow \frac{dC}{dm} = \frac{V - m\frac{dV}{dm}}{V^2}$

Practice Problems: 3.3: # 1-6, 8, 9
3.4: 1-6 are okay. Don't worry about remembering the vocab though
3.3 # 7

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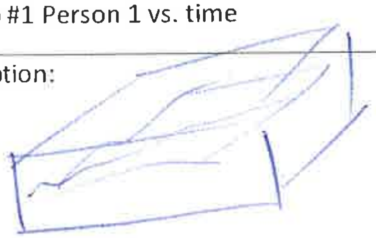
Scenario #1 Person 1 vs. time Volume vs time

Description:



Graph:

$\frac{dV}{dt}$ is fast to start

Equation:

$$V(t) = 5\sqrt{t} + 10$$

Differential Equation:

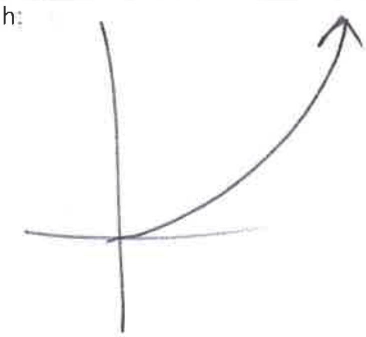
$$\frac{dV}{dt} = \frac{5}{2\sqrt{t}} \quad \frac{cm^3}{s}$$

Scenario #1 Person 2 vs. Any unit that is not time

Description: Volume vs $ cost

$\frac{dV}{dC}$

Graph:



Equation:	Differential Equations:
$V(p) = \frac{1}{12}p^2 + 1$	$\frac{dV}{dp} = \frac{1}{6}p$ $\frac{dV}{dp} \uparrow$ as $p \uparrow$

Example: Concentration is measure in amount of substance per unit volume. Imagine we start with a empty glass and add water at some variable rate and add sugar crystals at some other rate.

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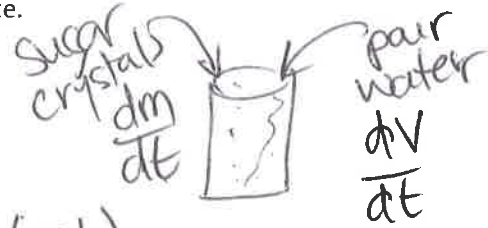

let V be volume of water (mL)

let m be mass of sugar crystals (g)

(a) let C be concentration (g/mL)

$$C = \frac{m}{V_{w} + V_{s}}$$

★ related variables

(b) $\frac{d}{dt}\left[C = \frac{m}{V}\right] \Rightarrow \frac{dC}{dt} = \frac{V\frac{dm}{dt} - m\frac{dV}{dt}}{V^2}$

★ related rates

Variables: $V, m, \frac{dC}{dt}, \frac{dm}{dt}, \frac{dV}{dt}$

(c) $\frac{d}{dm}\left[C = \frac{m}{V}\right] \Rightarrow \frac{dC}{dm} = \frac{V - m\frac{dV}{dm}}{V^2}$

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