Asymptotes: Horizontal and Otherwise

Goal:
- Can identify the vertical, horizontal, and slant asymptotes of a function
- Understand that asymptotes are the description of regular behaviour as something becomes infinite
- Gaining comfort to graph key characteristics of functions.

Terminology:
- Slant Asymptote
- Infinite Limit

Review: From grade 12 what is a horizontal and vertical asymptote?

Calculus: We want to amend the above description to have a calculus perspective:

\[
\lim_{x \to \pm \infty} f(x) = L \implies \text{horizontal asymptote}
\]

\[
\lim_{x \to c^-} f(x) = \pm \infty \quad \text{vertical asymptote}
\]
When we approach infinity with polynomials, we only care about relative growth. That is $x^2$ grows faster than $x$, and $x^3$ grows faster still.

\[
growth \quad O(x^n) > O(x^{n-1})
\]

So for large values of $x$, only the leading term really matters.

Example:

\[
\lim_{x \to \infty} \frac{6x^4 + 6x^2 - 100}{2x^4 - 49x^3 + 10000}
\]

degree of numerator is 4

\[
\Rightarrow \lim_{x \to \infty} \frac{6x^4}{2x^4} = 3
\]

Practice:

\[
\lim_{x \to -\infty} \frac{4x^5 - 17x^3 + 400x - 20^{10}}{0.001x^6 - x^5 + 200x^2}
\]

\[
\lim_{x \to -\infty} \frac{1}{x^6} = 0
\]
But what happens when the degree of the numerator is greater than the degree of the denominator?

**Example:**

\[
\lim_{{x \to \infty}} \frac{2x^3 - x^2 + 400x}{x^2 + x + 1}
\]

degree of top is 3

1 more than the bottom

\[
\frac{2x}{x^2 + 1} - \frac{3}{2}
\]

\[
\lim_{{x \to \infty}} \frac{2x - 3}{x^2 + x}
\]

\[
\lim_{{x \to \infty}} \frac{3x}{x^2 + x}
\]

Practice:

\[
\lim_{{x \to \infty}} \frac{2x^5 - 2x^4 - 10x^2 + 1}{x^4 + 1}
\]

degree of top is 1

more so we need \(x^2\) term is more

\[
\lim_{{x \to \infty}} \frac{7x - 2}{x^4}
\]

\[
\lim_{{x \to \infty}} (7x - 2) \rightarrow \text{slant asymptote}
\]

**Practice Problems:**

5.1: # Anything you feel is valuable (This section is Precalc 12 and early limit review)

5.2: # 1-3 (do what you need), 4, 6, 11

5.6: # 1-3

5.2 # 7-10
Desmos Asymptote Activity

I want you to find the equation to the horizontal and slant asymptotes by using Desmos to graph and compare the rational function to the equation to the asymptote as I showed in class.

Go to:

```
desmos.com/calculator/rhnw0r4upz
```

Find the equation to the horizontal/slant asymptote and graph the asymptote along with the graph.

1. \[
\frac{x^6 - 5x^3 - 100}{x^7 + 30x^6}
\]

2. \[
\frac{-x^5 + 100x^3 - 200}{0.01x^8 + 100}
\]
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Vertical asymptote
\[
\lim_{x \to c} \frac{p(x)}{q(x)} = \pm \infty \quad q(x) \to 0 \quad p(x) \to 0
\]

Asymptote

Horizontal asymptote
\[
\lim_{x \to \pm \infty} \frac{p(x)}{q(x)} = L
\]
When we approach infinity with polynomials, we only care about relative growth. That is $x^2$ grows faster than $x$, and $x^3$ grows faster still.

\[ \sigma(x^n) > \sigma(x^{n-1}) \]

So for large values of $x$, only the leading term really matters.

**Example:**

\[
\lim_{x \to \infty} \frac{6x^4 + 6x^2 - 100}{2x^3 - 49x^2 + 10000} = 3
\]

Horizontal asymptote

**Practice:**

\[
\lim_{x \to -\infty} \frac{4x^5 - 17x^3 + 400x - 20^{20}}{0.001x^6 - x^5 + 200x^2} = 0
\]

\[\text{Or}\]

\[
\lim_{x \to -\infty} \frac{4x^5}{0.001x^6} = \lim_{x \to -\infty} \frac{4}{0.001x} = 0
\]
But what happens when the degree of the numerator is greater than the degree of the denominator?

Example:

\[
\lim_{x \to \infty} \frac{2x^3 - x^2 + 400x}{x^2 + x + 1}
\]

\[
\lim_{x \to \infty} \frac{2x^3}{x^2 + x + 1} = \lim_{x \to \infty} \frac{2x}{1} = 0
\]

Practice:

\[
\lim_{x \to \infty} \frac{7x^5 - 2x^4 - 10x^2 + 1}{x^4 + 1}
\]

\[
\lim_{x \to \infty} \frac{7x^5}{x^4} = 7x - 2
\]

Practice Problems: 5.1: # Anything you feel is valuable (This section is Precalc 12 and early limit review)
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5.6: # 1-3

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2. \[ \frac{-x^6 + 100x^3 - 200}{0.01x^8 + 100} \]