2

Asymptotes: Horizontal and Otherwise

Goal:

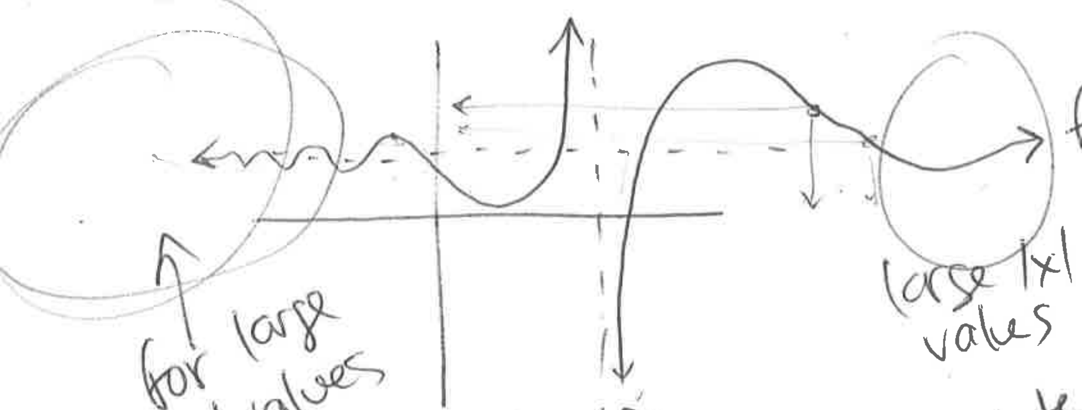
- Can identify the vertical, horizontal, and slant asymptotes of a function
- Understand that asymptotes are the description of regular behaviour as something becomes infinite
- Gaining comfort to graph key characteristics of functions.

Terminology:

- Slant Asymptote
- Infinite Limit

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$$

Review: From grade 12 what is a horizontal and vertical asymptote?



$$f(x) = \frac{p(x)}{q(x)}$$

for large |x| values we have a horizontal asymptote

what causes vertical asymptote zero in denominator AND numerator is NOT zero

Calculus: We want to ammend the above description to have a calculus perspective:

$$\lim_{x \to \pm\infty} f(x) = L \Rightarrow \text{horizontal asymptote}$$

$$\lim_{x \to c^{-/+}} f(x) = \pm\infty$$

vertical asymptote

When we approach infinity with polynomials, we only care about relative growth. That is x^2 grows faster than x, and x^3 grows faster still.

growth

$$O(x^n) > O(x^{n-1})$$

So for large values of x, only the leading term really maters

Example:

degree of numerator is 4

$$\lim_{x\to\infty} \frac{6x^4 + 6x^2 - 100}{2x^4 - 49x^3 + 10000}$$

divisor is degree 4

$$\Rightarrow \lim_{x\to\infty} \frac{6x^4}{2x^4} = 3$$

Practice:

$$\lim_{x\to-\infty} \frac{4x^5 - 17x^3 + 400x - 20^{20}}{\underbrace{0.001x^6 - x^5 + 200x^2}_{\text{large}}}$$

$$\lim_{x\to-\infty} \frac{1}{x^6} = 0$$

But what happens when the degree of the numerator is greater than the degree of the denominator?

Example:

degree of top is 3 1 more than the bottom

$$\lim_{x\to\infty}\frac{2x^3-x^2+400x}{x^2+x+1}$$

less than degree 2

$$\lim_{x\to\infty}\frac{2x^3-x^2}{x^2+x}$$

$$\begin{array}{r} 2x \quad -3 \\ x^2+x\,\overline{)\,2x^3-x^2} \\ -(2x^3+2x^2) \\ \hline -3x^2 \\ -(-3x^2-3x) \\ \hline +3x \end{array}$$

→ remainder

$$\lim_{x\to\infty}\boxed{2x-3}+\cancel{\frac{3x}{x^2+x}}\to 0$$

Practice:

$$\lim_{x\to\infty}\frac{7x^5-2x^4+10x^2+1}{x^4+1}$$

less than degree 4

degree of top is 1 more so we need x^3 term (1 more)

$$\boxed{7x-2}$$

$$=\lim_{x\to\infty}\frac{7x^5-2x^4}{x^4}$$

$$=\lim_{x\to\infty} 7x-2$$ → slant asymptote

Practice Problems: 5.1: # Anything you feel is valuable (This section is Precalc 12 and early limit review)

5.2: # 1-3 (do what you need), 4, 6, 11

5.6: # 1-3

🌶️ 5.2 # 7-10

Desmos Asymptote Activity

I want you to find the equation to the horizontal and slant asymptotes by using Desmos to graph and compare the rational function to the equation to the asymptote as I showed in class.

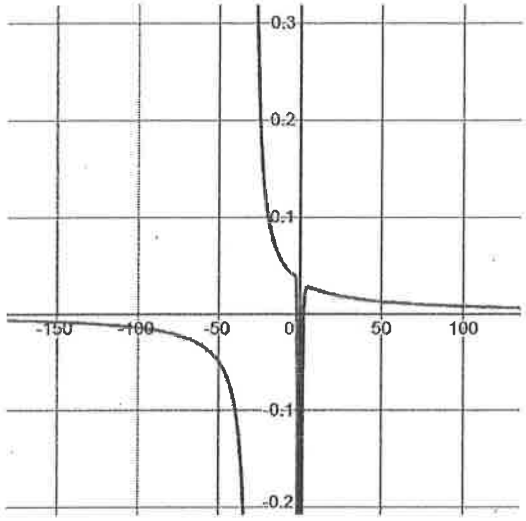
Go to:

desmos.com/calculator/rhnw0r4upz

Find the equation to the horizontal/slant asymptote and graph the asymptote along with the graph.

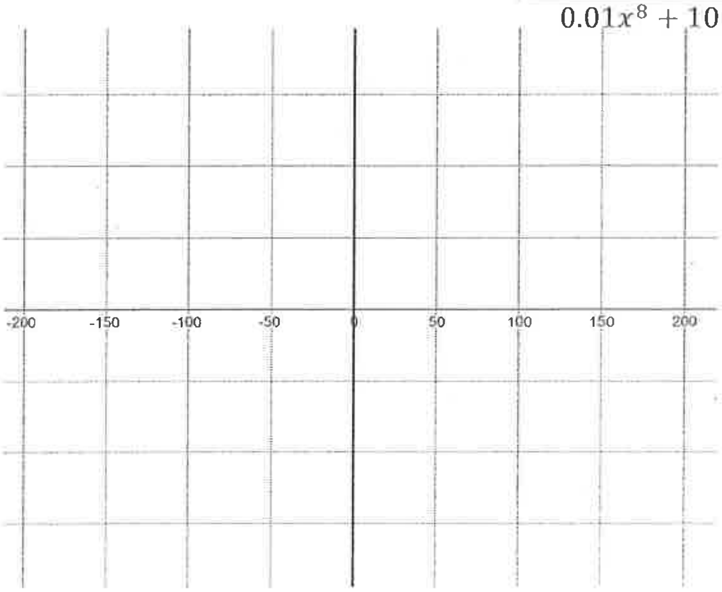
1.

$$\frac{x^6 - 5x^3 - 100}{x^7 + 30x^6}$$



2.

$$\frac{-x^5 + 100x^3 - 200}{0.01x^8 + 100}$$



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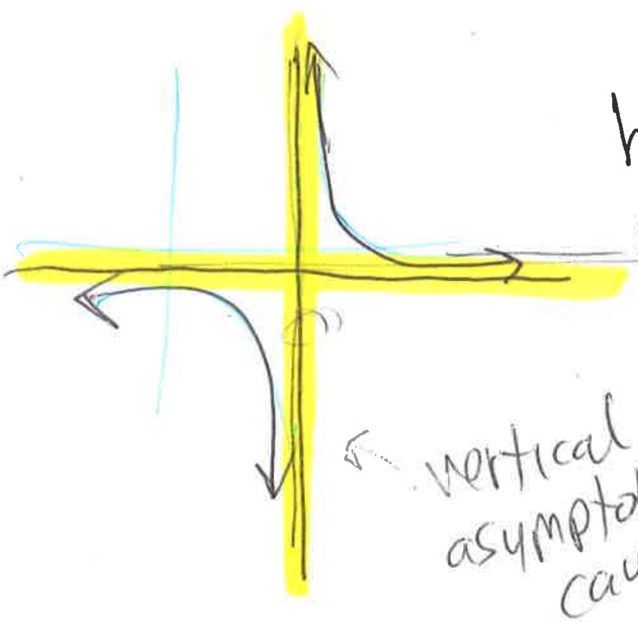

horizontal asymptote caused by x getting big and y approach something

vertical asymptote caused by denominator is zero AND numerator not 0

Calculus: We want to ammend the above description to have a calculus perspective.

Vertical asymptote

$$\lim_{x \to c} \frac{p(x)}{q(x)} = \pm\infty \qquad q(x) \to 0 \qquad p(x) \not\to 0$$

↑ asymptote

Horizontal asymptote

$$\lim_{x \to \pm\infty} \frac{p(x)}{q(x)} = L$$

← horizontal asymptote

When we approach infinity with polynomials, we only care about relative growth. That is x^2 grows faster than x, and x^3 grows faster still.

growth

$$O(x^n) > O(x^{n-1})$$

So for large values of x, only the leading term really maters

Example:

not going to matter

look @ divisor

$$\lim_{x\to\infty} \frac{6x^4 + 6x^2 - 100}{2x^4 \underbrace{- 49x^3 + 10000}_{\text{not goin to matter}}}$$

$$\Rightarrow \lim_{x\to\infty} \frac{6x^4}{2x^4} = 3$$

horizontal asymptote

Practice:

not goin to matter

$$\lim_{x\to-\infty} \frac{4x^5 - 17x^3 + 400x - 2020}{0.001x^6 - x^5 + 200x^2}$$

$$\lim_{x\to-\infty} \frac{1}{0.001x^6} = 0$$

$$\text{OR } \lim_{x\to-\infty} \frac{4x^5}{0.001x^6} = \lim_{x\to-\infty} \frac{4}{0.001x}$$

$$= 0$$

But what happens when the degree of the numerator is greater than the degree of the denominator?

Example:

$$\lim_{x\to\infty}\frac{2x^3-x^2+400x}{x^2+x+1}$$

→not going to matter

$$\lim_{x\to\infty}\frac{2x^3-x^2}{x^2+x}=\lim_{x\to\infty}\frac{\cancel{x^2}(2x-1)}{\cancel{x}(x+1)}$$

$$=\lim_{x\to\infty}\frac{2x^2-x}{x+1}$$

$$\begin{array}{r}
2x\ \ -3\\
x+1\,\big)\ 2x^2\ \ -x\\
-(2x^2+2x)\\
\hline
-3x\\
-3x\ \ -3
\end{array}$$

→remainder

$$=\lim_{x\to\infty}\boxed{2x-3}\ \cancel{\frac{-3}{x+1}}\ 0$$

slant asymptote

Practice:

$$\lim_{x\to\infty}\frac{7x^5-2x^4-10x^2+1}{x^4+1}$$

degree top is 1 ~~less~~ more than bottom need 1 more term, x^3

less than x^4 not going to matter

$$\lim_{x\to\infty}\frac{7x^5-2x^4}{x^4}$$

$$\boxed{7x-2}$$

$$=\lim_{x\to\infty}7x-2$$

slant asymptote

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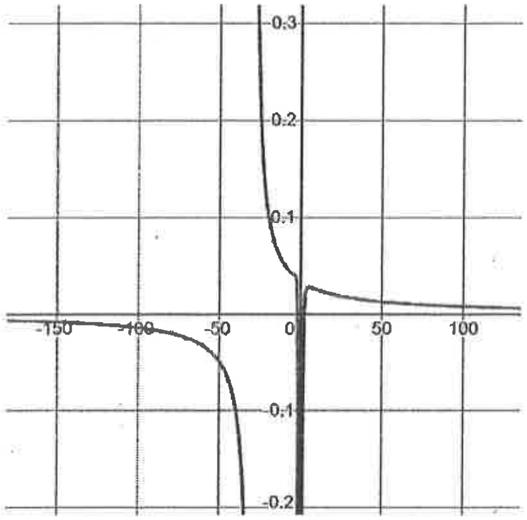

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