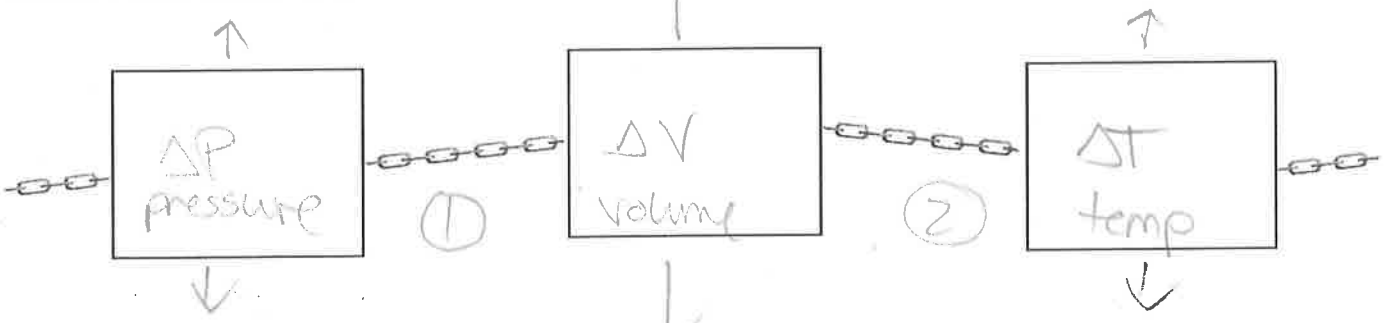
Chain Rule

Goal: • Can describe chain rule as the measure of how much something two steps down changes. • Can use chain rule along with the other derivative rules to find derivatives.
Terminology: • Chain Rule

Consider a chain that has three blocks connected like illustrated below:



If we move the block furthest to the left, it will cause the black on the far right to move a related amount. We are interested in measuring that rate of change. It helps to use units

we can look at rates of change

① $\frac{\Delta V}{\Delta P}$ and ② $\frac{\Delta T}{\Delta V}$

we want to consider how much ΔT is when we know ΔP

$$\frac{\Delta T}{\Delta P} = \frac{\Delta T}{\Delta V} \cdot \frac{\Delta V}{\Delta P}$$

let $\Delta P = \Delta x$

$\Delta V = \Delta f(x) = \Delta u$

$\Delta T = \Delta g(f(x)) = \Delta g(u)$

$$\Rightarrow \frac{\Delta g}{\Delta x} = \frac{\Delta g}{\Delta u} \cdot \frac{\Delta u}{\Delta x} \xrightarrow{\text{limit}} \boxed{\frac{dg}{dx} = \frac{dg}{du} \cdot \frac{du}{dx}}$$

This is chain rule!

Example: If $A(x) = f(g(x)) + x^2$ and we know $f(2) = 4, f'(6) = 5, g(2) = 6, g'(2) = -3$. Determine $A'(2)$. (Alternatively, you could write it as $\frac{d}{dx}A(x)|_{x=2}$)

$$A'(x) = \frac{d}{dx}\left(f(g(x)) + x^2\right)$$

$$= f'(g(x)) \cdot g'(x) + 2x$$

$$A'(2) = f'(g(2)) \cdot g'(2) + 2(2)$$

$$= f'(6) \cdot (-3) + 4$$

$$= -15 + 4 = -11$$

Example: If $y = \sqrt{u+3}$ and $u = \frac{1}{w^2}$ and $w = x^4 - 4x^3 + 8x$. Find $\frac{dy}{du}, \frac{dy}{dw}$, and $\frac{dy}{dx}$

$$\frac{dy}{du} = \frac{d}{du}\left(\sqrt{u+3}\right) = \frac{1}{2}(u+3)^{-1/2}$$

$$\frac{dy}{dw} = \frac{dy}{du} \cdot \frac{du}{dw} = \frac{1}{2}(u+3)^{-1/2} \cdot \frac{d}{dw}(w^{-2})$$

$$= -\frac{1}{2}(u+3)^{-1/2}(2w^{-3})$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dw} \cdot \frac{dw}{dx} = -(u+3)^{-1/2} w^{-3} \cdot \frac{d}{dx}(x^4 - 4x^3 + 8x)$$

$$= \frac{4x^3 - 12x^2 + 8}{-w^3 (u+3)^{1/2}}$$

Practice Problems: 2.6: # 1-5 (do what you need), 6 (at least every other), 7-11

12, Problems Plus (page 102)