First Derivative Applications Chapter Test: Version A

Name: ______________________ Date: February 4, 2020

1. Consider the function

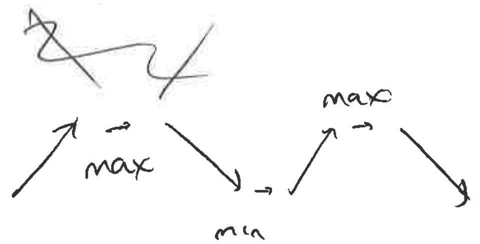
$$f(x) = -x^4 + 2x^3 + 5x^2 - 2$$

(a) (4 points) Detemine all the extrema of f (absolute and local). Justify your explanation.

$$f'(x) = -4x^3 + 6x^2 + 10x$$
$$= -2x(2x^2 - 3x - 5)$$
$$= -2x(2x - 5)(x + 1)$$

-1 0 2.5

$f'(x)$ + 0 − 0 + 0 −

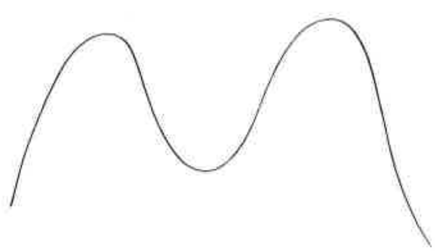


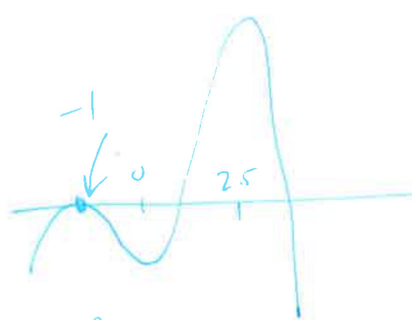
$f(-1) = 0$ local max

$f(0) = -2$ local min

$f(2.5) = 21.4375$ ab max

(b) (4 points) Determine all of the zeros of f. Explain why you won't find 4 zeros.





$$x_{n+1} = \frac{-f(x_n)}{f'(x_n)} + x_n$$

only 3 zeros b/c

$x_0 = -1 \rightarrow x = -1$

$x_0 = 1 \rightarrow x = 0.58578....$

$x_0 = 5 \rightarrow x = 3.4142..$

2. Consider a **Continuous** function g that has the following values of g' given below. In the table, Pos. means positive and Neg. means negative.

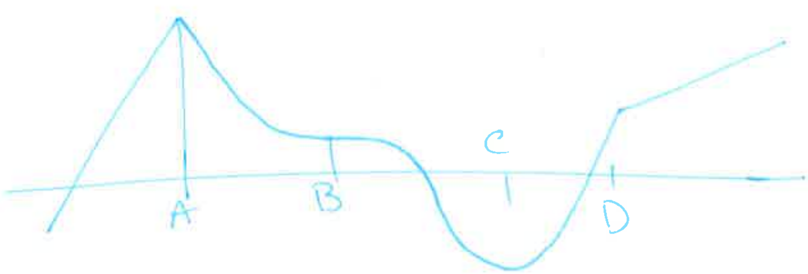
x	$x < A$	A	$A < x < B$	B	$B < x < C$	C	$C < x < D$	D	$x > D$
$g'(x)$	Pos.	Undefined	Neg.	0	Neg.	0	Pos.	Undefined	Pos.

(a) (2 points) For what value of x will g have a local extrema? Justify your explanation.

local max @ $x = A$ (+ to −)

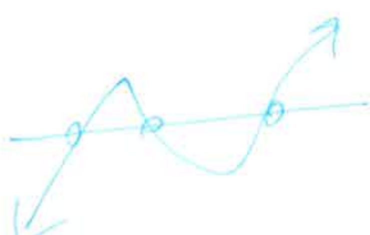
local min @ $x = C$ (− to +)

(b) (2 points) Sketch a rough graph of what g could look like.

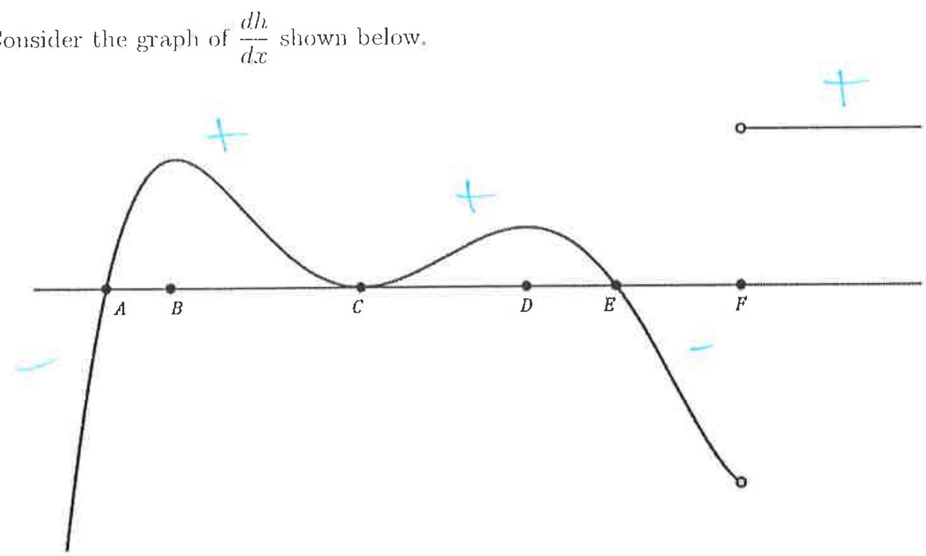


(c) (1 point) What is the maximum number of zeros you expect g to have? Just a brief justification is necessary.

3 b/c curve goes up / down / up like a cubic



3. (2 points) Consider the graph of $\frac{dh}{dx}$ shown below.



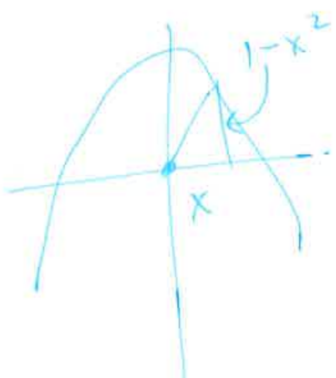
For what values of x does h have a local extrema? Justify your explanation.

min @ $x = A$ ($-$ to $+$)

max @ $x = E$ ($+$ to $-$)

min @ $x = F$ ($-$ to $+$)

4. (3 points) What is the shortest distance between the curve $y = 1 - x^2$ and the origin?



$$d(x) = \sqrt{x^2 + (1-x^2)^2}$$

$$d(x) = \sqrt{x^4 - x^2 + 1}$$

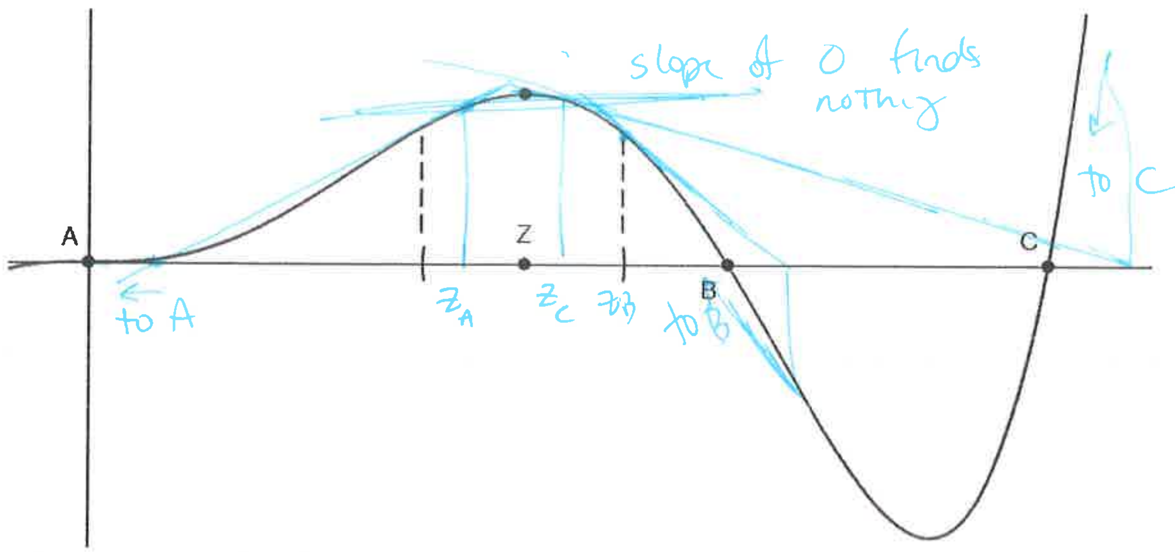
$$d'(x) = \frac{4x^3 - 2x}{2\sqrt{x^4 - x^2 + 1}} = 0$$

$$4x^{\cancel{3}\,2} - 2x = 0$$

$$x = \frac{1}{\sqrt{2}} \Rightarrow d = \sqrt{\frac{1}{4} - \frac{1}{2} + 1}$$

$$= \sqrt{1 - 1/4} = \sqrt{3}/2$$

5. (4 points) Using Newton's method, consider the curve and points shown below.



Show that choosing a value of z_0 around $x = Z$ (inside the wedge) could lead to finding any of the zeros or nothing.

slope 0 finds nothing $\rightarrow$ ~~z~~ $z_0 = Z$

if $z_A < Z$ then it finds A

at edge of wedge ~~of~~ on right z_B finds B

and just right of Z will find C

6. (2 points) If the cost to produce q items (each in units of a thousand) is

$$C(q) = q^3 - 6q^2 + 12q$$

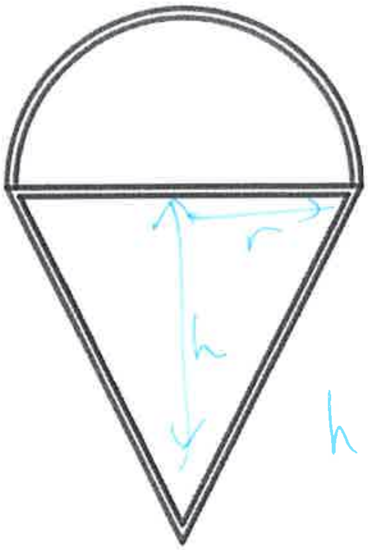
What is the amount that should be produced to minimize the average cost?

$\frac{C(q)}{q} \quad \frac{d}{dq}\left(q^2 - 6q + 12\right)$

$0 = 2q - 6 \Rightarrow \boxed{q = 3}$ 3000 units

7. (4 points) A neon ice cream cone is shown below, made by putting a semicircle on top of a triangle. If the total length of neon light used is fixed, we want to determine the dimensions of the light that maximize the total area.

IMPORTANT: Once you have taken a derivative and set it equal to 0 you are done! Do NOT try and solve for any of the dimensions (the algebra is not what I am assessing).



$$P = 2r + 2\sqrt{r^2+h^2} + \pi r$$

$$A = \frac{\pi r^2}{2} + rh$$

$$h = \sqrt{\left(\frac{P-2r-\pi r}{2}\right)^2 - r^2}$$

$$A = \frac{\pi r^2}{2} + r\sqrt{\left(\frac{P-2r-\pi r}{2}\right)^2 - r^2}$$

$$A' = \pi r + \sqrt{\left(\frac{P-2r-\pi r}{2}\right)^2 - r^2} + \frac{r\left[\left(\frac{P-2r-\pi r}{2}\right)(-2-\pi) - 2r\right]}{2\sqrt{\left(\frac{P-2r-\pi r}{2}\right)^2 - r^2}}$$

$$= 0$$

8. (1 point (bonus)) Consider a sequence defined as

$$x_{n+1} = 2.5x_n(1 - x_n)$$

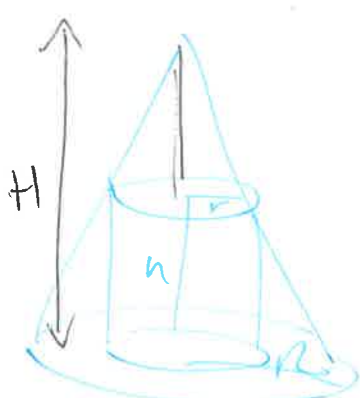
Let L be a real number defined as

$$L = \lim_{n\to\infty} x_n$$

Determine the value of L.

$L = 0.6 = 3/5$

9. (1 point (bonus)) Determine the radius of the largest cylinder that can be inscribed inside a cone with radius R and height H.



$\frac{R}{H} = \frac{r}{H-h}$ max $V = \pi r^2 h$

$r = \frac{R}{H}(H-h)$ $V = \pi h \left(\frac{R}{H}(H-h)\right)^2$

$V'(h) = \pi\left(\frac{R}{H}(H-h)\right)^2 + 2\pi h \frac{R}{H}(H-h)(-1) = 0$

$\left(\frac{R}{H}(H-h)\right)^{\cancel{2}} = 2h\cancel{\left(\frac{R}{H}(H-h)\right)}$

$R - \frac{Rh}{H} = 2h$

$R = 2h + \frac{R}{H}h = h\left(2 + \frac{R}{H}\right)$ $h = \frac{R}{2 + R/H}$

$r = \frac{R}{H}\left(H - \frac{RH}{2H+R}\right) = R - \frac{R^2}{2H+R}$

$= \frac{2HR}{2H+R}$