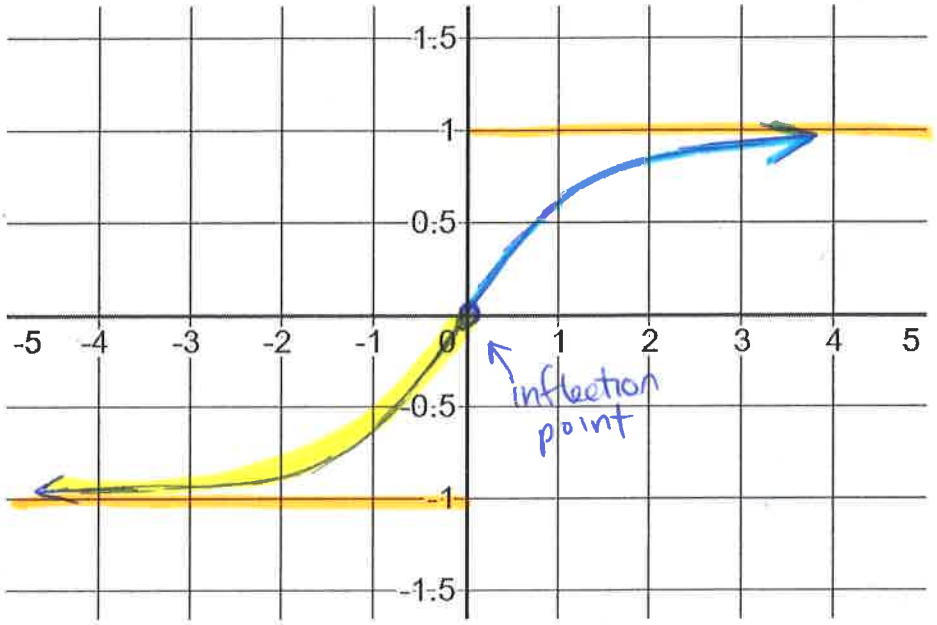
Curve Sketching Practice

Sketch the graphs with correct intercepts, asymptotes, local extrema and inflection points.

1.

$$y = \frac{x}{\sqrt{x^2+2}}$$



$y=0 \Rightarrow x=0$

$y = x(x^2+2)^{-1/2}$

$y' = (x^2+2)^{-1/2} - x^2(x^2+2)^{-3/2}$

$= (x^2+2)^{-3/2}\left(x^2+2-x^2\right)$

$= 2(x^2+2)^{-3/2}$

$y' = 0$ never

$\Rightarrow$ no extrema

$y'' = -3(x^2+2)^{-5/2}(2x)$

$y''=0 \Rightarrow x=0$

x		0	
y''	$+$	0	$-$
	$\cup$	to	$\cap$

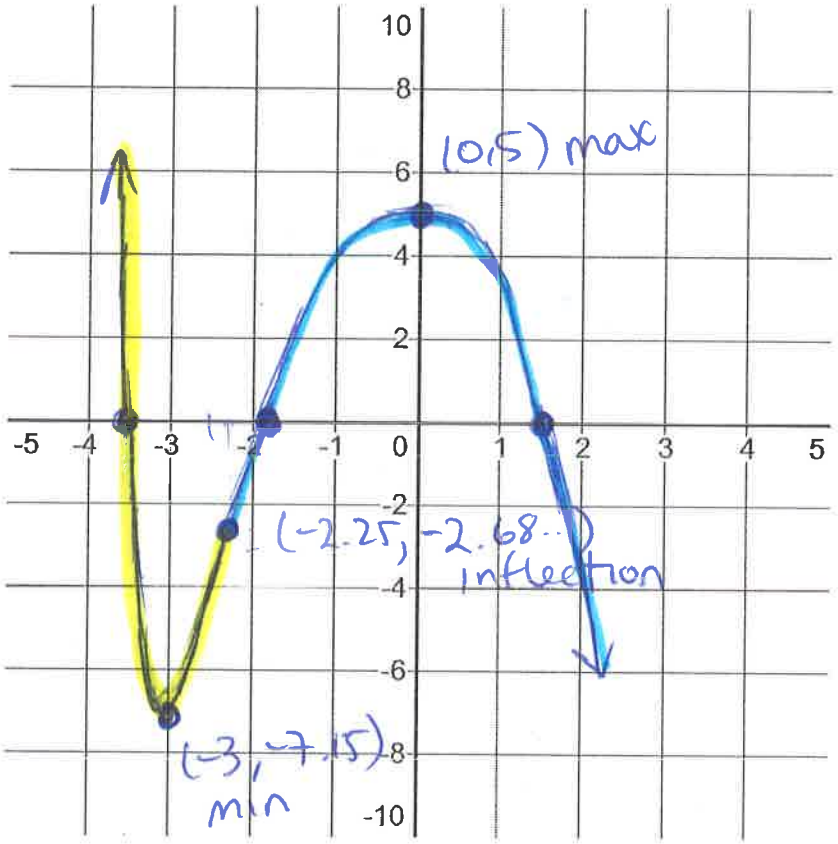
HA $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+2}} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2}} =$ Am

$= \lim_{x\to\infty} \frac{x}{|x|} = 1$

$\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+2}} = \lim_{x\to-\infty} \frac{x}{|x|} = -1$

2.

$$y = -\frac{1}{5}x^5 - \frac{3}{4}x^4 + 5$$



$y' = -x^4 - 3x^3$

$= -x^3(x+3)$

$x = 0$ and -3

both change sign

extrema @ $x=0$ and -3

$y'' = -4x^3 - 9x^2$

$= -x^2(4x+9)$

$x = 0$ doesn't change sign

$x = -2.25$ inflection

zeros: use Newtons method

$$x = A - \frac{\frac{A^5}{5} + \frac{3}{4}A^4 - 5}{-A^4 - 3A^3}$$

find $x = 1.478...$

$= -3.601...$

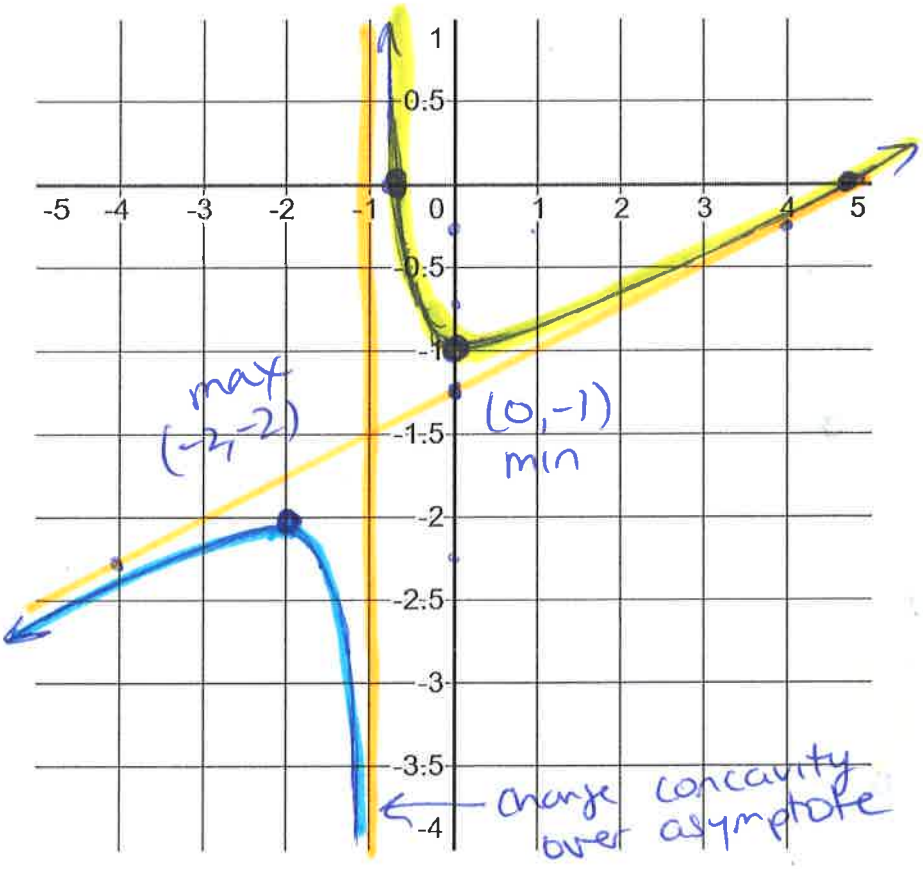
$= -1.923...$

★ note the squashed parabola shape @ $x=0$

where $y \approx -\frac{3}{4}x^4 + 5$ ↑ a quartic

3.

$$y = \frac{0.25x^2 - x - 1}{x + 1}$$



slant $\frac{1}{4}x - 5/4$

$x+1 \overline{)\frac{1}{4}x^2 - x}$

$-(\frac{1}{4}x^2 + \frac{1}{4}x)$

$-5/4x$

slant asymptote $y = \frac{1}{4}x - \frac{5}{4}$

$y' = \frac{(\frac{1}{2}x - 1)(x+1) - (\frac{1}{4}x^2 - x - 1)}{(x+1)^2}$

$= \frac{\frac{1}{2}x^2 - \frac{1}{2}x \cancel{- 1} - \frac{1}{4}x^2 + x \cancel{+ 1}}{(x+1)^2}$

$= \frac{\frac{1}{4}x^2 + \frac{1}{2}x}{(x+1)^2} = 0$

$\Rightarrow \frac{1}{4}x(x+2) = 0$

$x = 0$ and -2

extrema

zeros $0.25x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1 + 4(\frac{1}{4})}}{2(\frac{1}{4})}$

$= \frac{1 \pm \sqrt{2}}{1/2} = 2 \pm 2\sqrt{2}$

$\approx 4.828\ldots, -0.828\ldots$

$y'' = \frac{(\frac{1}{2}x + \frac{1}{2})(x+1)^2 - 2(x+1)(\frac{1}{4}x^2 + \frac{1}{2}x)}{\sim}$

$\Rightarrow y'' = 0$ so $\frac{1}{2}(x+1)^3 - 2x(x+1)(\frac{1}{4}x + \frac{1}{2}) = 0$

$(x+1)\left[\frac{1}{2}(x+1)^2 - 2x(\frac{1}{4}x + \frac{1}{2})\right] = 0$

$\frac{1}{2}\left[x^2 + 2x + 1\right] - \frac{1}{2}x^2 - x = 0$

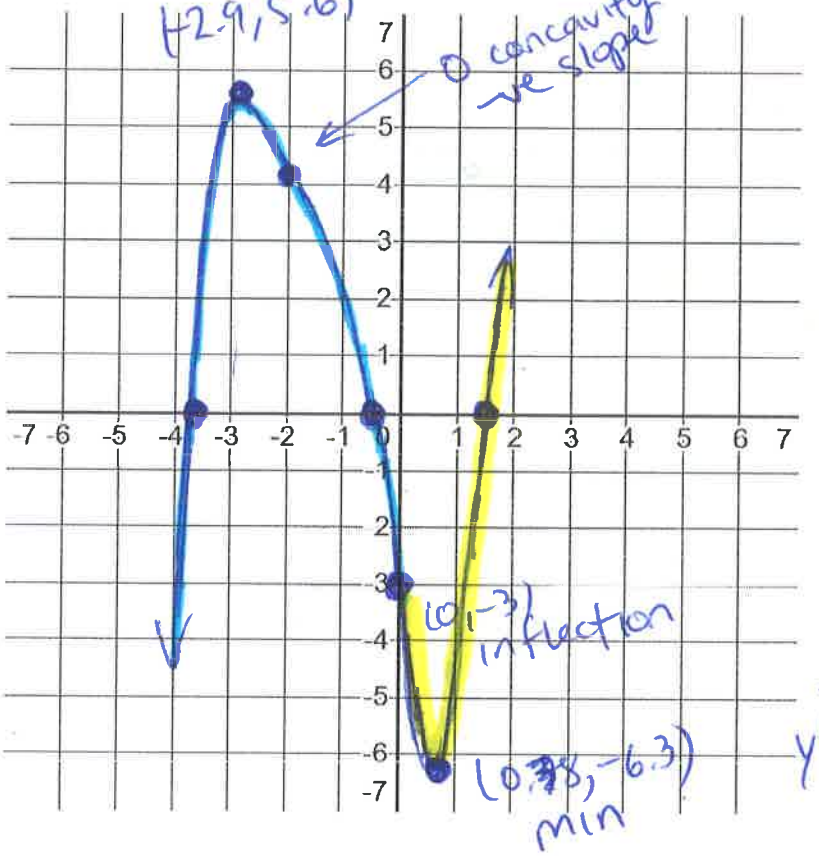
no zeros

$x = -1$

only inflection point

4.

$$y = \frac{3}{20}x^5 + x^4 + 2x^3 - 6x - 3$$



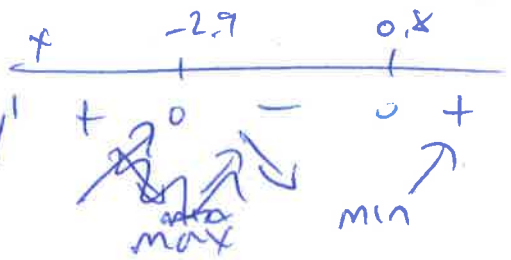
when $y' = 0$ use Newtons

$y'' = 3x^3 + 12x^2 + 12x$

$x = -\frac{y'(A)}{y''(A)} + A$

find $x = 0.789...$ both extrema

$= -2.907...$



Zeros using Newtons:

$y' = \frac{3}{4}x^4 + 4x^3 + 6x^2 - 6$

$x = -\frac{y(A)}{y'(A)} + A$

find $x = -0.539...$

$= -3.783...$

$= 1.459...$

$y'' = 3x(x^2 + 4x + 4)$

$= 3x(x+2)^2 = 0$

$\Rightarrow x = 0, -2$

↑ doesn't change sign NOT inflection