Derivative Rules Chapter Test: Version B

Name: ______________________________ Date: November 20, 2019

1. Given the following functions state the rule general derivative rules being used and determine dy/dx for each.

   (a) (2 points) $y = x^5 - 7x^3$

   $$\frac{dy}{dy} = 5x^4 - 21x^2$$

   power $\frac{d}{dx} x^n = nx^{n-1}$

   sum/diff. $\frac{d}{dx}(f \pm g) = f' \pm g'$

   (b) (2 points) $y = (2\sqrt{x} + x) \cdot (x^2 - 3)$

   $$\frac{dy}{dx} = \left(\frac{1}{\sqrt{x}} + 1\right)(x^2 - 3) + (2x)(2\sqrt{x} + x)$$

   prod. $(fg)' = f'g + g'f$

   (c) (2 points) $y = \dfrac{3x + 5}{x^2 - 5x + 6}$

   $$\frac{dy}{dx} = \frac{3(x^2 - 5x + 6) - (2x - 5)(3x + 5)}{(x^2 - 5x + 6)^2}$$

   quotient. $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

(d) (2 points) $y = \left(x^6 - 3x^4\right)^{4/3}$

$$\frac{dy}{dx} = \frac{4}{3}(x^6 - 3x^4)^{1/3} \cdot (6x^5 - 12x^3)$$

chain $\quad (f(g(x)))' = f'(g(x)) \cdot g'(x)$

2. (a) (2 points) State the definition of the derivative of $y = f(x)$ and what it means for f to be differentiable at a point.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

differentiable ~~means~~ @ $x = c$ means

$f'(c)$ exists

(b) (2 points) Use the definition of the derivative to show that $\frac{d}{dx}\left(\sqrt{2x+1}\right) = \frac{1}{\sqrt{2x+1}}$

$$\lim_{h \to 0} \frac{\sqrt{2x+2h+1} - \sqrt{2x+1}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x} + 2\cancel{h} + \cancel{1} - \cancel{2x} - \cancel{1}}{\cancel{h}\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}$$

$$= \frac{2}{\sqrt{2x+1} + \sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$

(c) (2 points) Use the definition of the derivative to show that the following function is not differntiable at the point $x = 3$.

$$f(x) = \begin{cases} (x-2)^2 & x \neq 3 \\ 4 & x = 3 \end{cases}$$

$$\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$\lim_{x \to 3} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{h \to 0} \frac{((3+h)-2)^2 - 4}{h}$$

$\frac{0}{0}$ ✓ $\frac{3}{0}$

$$= \lim_{h \to 0} \frac{1 + 2h + h^2 - 4}{h}$$

$$= \lim_{h \to 0} \frac{-3 + 2h + h^2}{h} = DNE$$

b/c the top goes to -3 and the bottom to 0 always have a large # divided by some thing small.

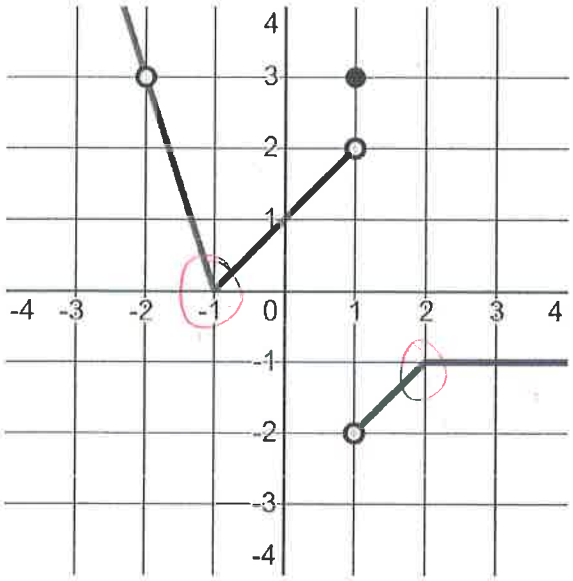
(d) (2 points) Use the definition of the derivative to show why Power Rule is true.

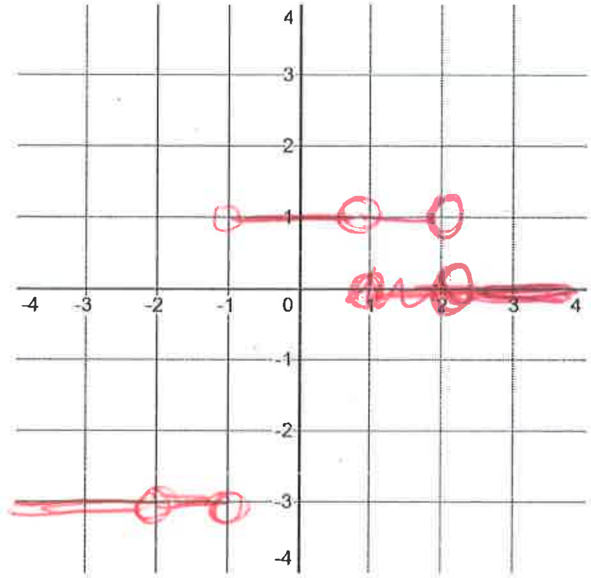
$$\lim_{h \to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h \to 0} \frac{\cancel{x^n} + nx^{n-1}\cancel{h} + \ldots + h^n - \cancel{x^n}}{\cancel{h}}$$

$$= \lim_{h \to 0} nx^{n-1} + \cancel{k}^{\to 0} + \ldots + \cancel{h}^{n-1 \to 0}$$

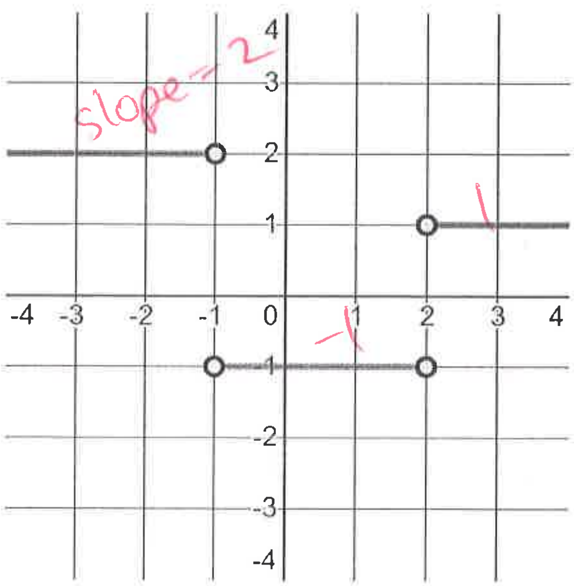
$$= nx^{n-1}$$

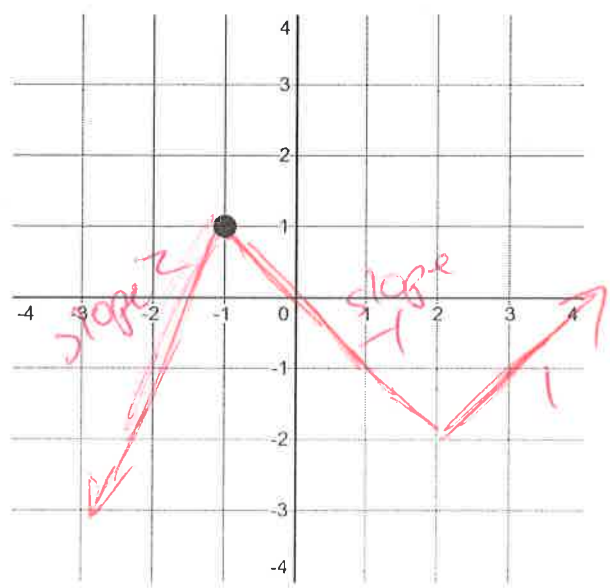
3. (a) (2 points) Given the graph of f below, graph f'.





(b) (2 points) Given the graph of dg/dx below, graph g. Make g continuous and pass through the indicated point $(-1, 1)$





4. Given the following relations determine the indicated derivative.

(a) (2 points) Find dy/dx given that

$$yx + y^3 = x^2$$

$$\frac{d}{dx}\left(yx+y^3\right) = \frac{d}{dx}x^2$$

$$\frac{dy}{dx}\cdot x + y + 3y^2\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{2x-y}{3y^2+x}$$

(b) (2 points) Find dy/dw given that

$$y = \frac{\boxed{(x+2w)^4}^{\,F}}{wx}$$

$=F'$

$$\frac{d}{dw}\left(\frac{(x+2w)^4}{wx}\right) \Rightarrow \left[4(x+2w)^3\left(\frac{dx}{dw}+2\right)wx\right]$$

$$\frac{dy}{dx} = \frac{dx}{dw}$$

$$\frac{d}{dw}(xw) = \frac{dx}{dw}\cdot w + x$$

$$\frac{dy}{dw} = \frac{4xw(x+2w)^3\left(\frac{dx}{dw}+2\right) - \left(\frac{dy}{dw}w+x\right)(x+2w)^4}{w^2x^2}$$

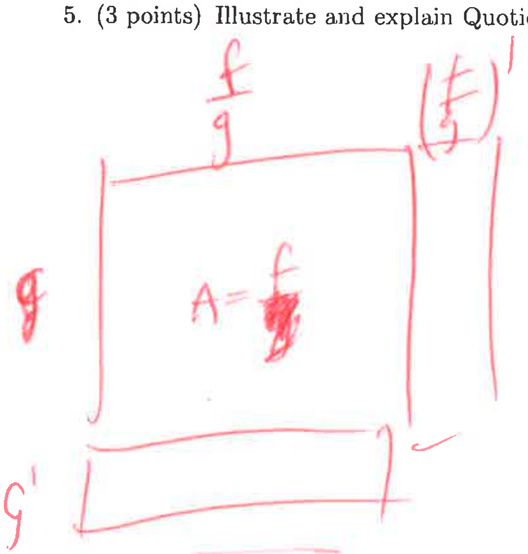
(c) (3 points) Find d^2y/dx^2 given that

$$y = f(g(u))$$

$$\frac{dy}{dx} = f'(g(u)) \cdot g'(u) \cdot u'$$

$$\frac{d^2y}{dx^2} = f''(g(u)) \cdot [g'(u) \cdot u']^2 + f'(g(u)) \cdot g''(u) \cdot (u')^2 + f'(g(u)) \cdot g'(u)(u'')$$

5. (3 points) Illustrate and explain Quotient Rule.



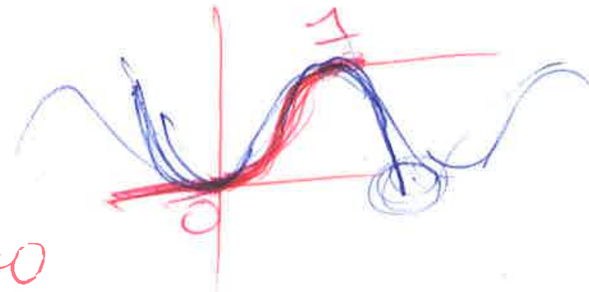
$$f' = \boxed{\left(\frac{f}{g}\right)'} g + g'f \swarrow \text{change in area}$$

$$\left(\frac{f}{g}\right)' = \frac{gf'}{g^2} - \frac{g'f}{g} = \frac{f'g - g'f}{g^2}$$

6. (2 points (bonus)) Determine $g(x)$ such that f is differentiable everywhere.

$$f(x) = \begin{cases} 0 & x < 0 \\ g(x) & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

$g'(0) = 0 \qquad g'(1) = 0$



$g(x) = Ax^2(x - b) \qquad g(0) = 0$

$g'(x) = 2Ax(x - b) + Ax^2$

$g'(0) = 0$ ✓ $\qquad g'(1) = 2A(1-b) + A$

$= 3A - 2Ab = 0$

$\Rightarrow b = 3/2$

$g(x) = Ax^2(x - 3/2)$

$g(1) = A(-1/2) = 1 \qquad A = -2$

$g(x) = -2x^2(x - 3/2)$