

The Derivative

Goal:

- Can use proper derivative notation to describe the slope of a curve
- Can determine if a function is differentiable based on its graph
- Understands that derivative is just another word for slope.

Terminology:

- Differentiable

Review: Write the slope of the tangent line to the curve $f(x)$ at the point $x = c$.

$$\lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h} \quad \text{or} \quad \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

in general at x

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \left| \text{The Derivative} \right|$$

Now that we are comfortable using this limit, we are going to define this slope at the general point x the derivative.

Notation: There are two ways we are going to talk about the derivative

1. Leibnitz's DX Notation

Slope = $\frac{\Delta y}{\Delta x} \rightarrow$ derivative

↑
"Delta"

The function NOT a fraction

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

↑ dy by dx

← what changes

the derivative of y with respect to x

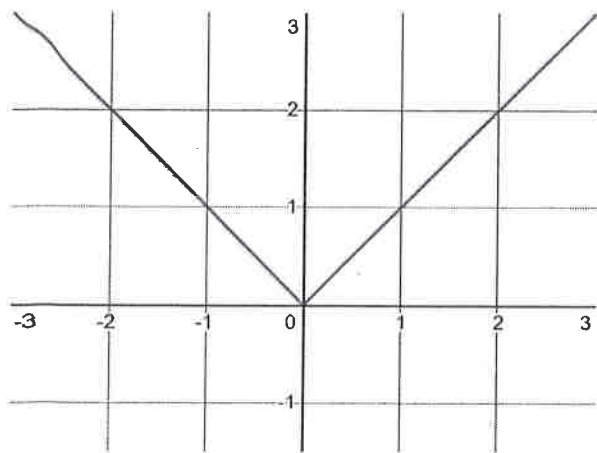
2. Newton's Prime Noation

if $y = f(x)$ then the derivative of y is y' or f'

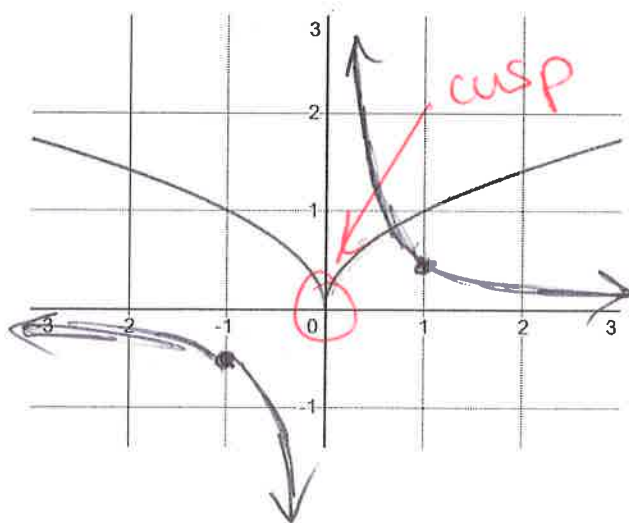
y' prime f' prime

Graph the derivatives of the following functions

1. $f(x) = |x|$



2. $g(x) = \sqrt{|x|}$



→ The derivative is not defined at $x=0$

→ Because there is an asymptote use limit definition @ $x=0$

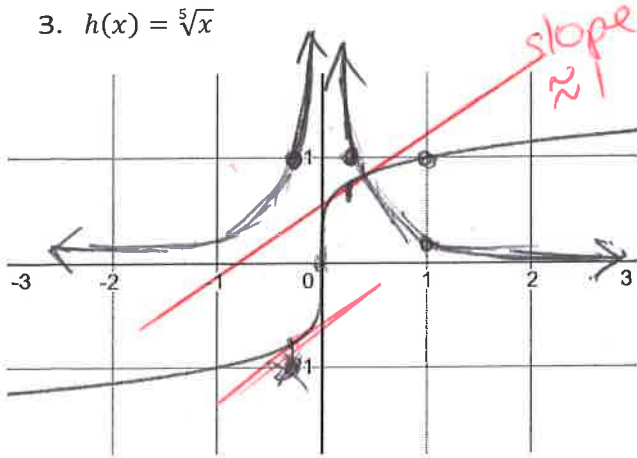
$$\lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{|h|} - 0}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{h}}{h}^*$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{h}}$$

DNE

3. $h(x) = \sqrt[5]{x}$

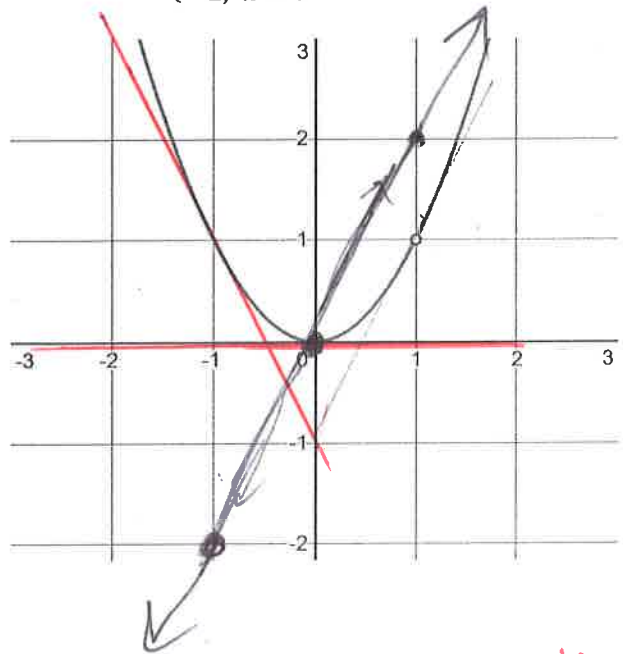


→ no derivative at $x=0$

→ we have a vertical tangent line at $x=0$

4.

$$k(x) = \begin{cases} x^2, & x \neq 1 \\ 2, & x = 1 \end{cases}$$



tell me what is the slope @ $x=c, c \neq 1$

$$\lim_{x \rightarrow c} \frac{x^2 - c^2}{x - c} = 2c$$

$$k'(x) = 2x$$

@ $x=1$

$$\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

b/c $f(1) = 2$
There will be problems

All the graphs are differentiable (the derivative exists) at every point except one



