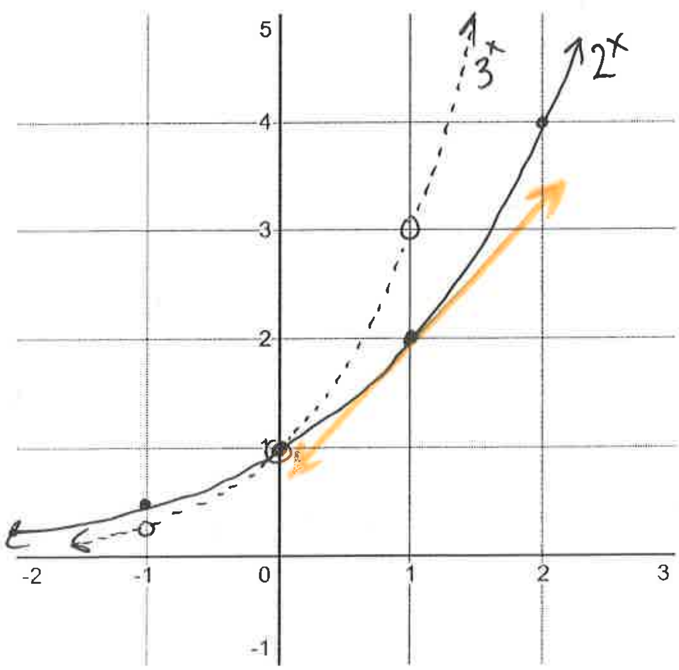
Derivative of the Exponential

Goal: • Can take the derivative of e^x with other derivative rules • Understands that e is built to be its own derivative.
Terminology: • Euler's Number
Reminder: • Make-up Test on Thursday March 5th after school

Review: Sketch the function $f(x) = 2^x$ and $g(x) = 3^x$



On the board try to determine $f'(x)$ and $g'(x)$

$$\lim_{h\to 0} \frac{2^{x+h}-2^x}{h} = \lim_{h\to 0} 2^x\left(\frac{2^h-1}{h}\right) = 2^x \lim_{h\to 0} \frac{2^h-1}{h}$$

$$= 2^x(0.693\ldots)$$

$$\lim_{h\to 0} \frac{3^{x+h}-3^x}{h} = \lim_{h\to 0} 3^x\left(\frac{3^h-1}{h}\right) = 3^x \lim_{h\to 0} \frac{3^h-1}{h}$$

$$= 3^x(1.0986\ldots)$$

From the video we are motivated to find a number $a \in (2.3)$ such that

$$\frac{d}{dx}a^x = a^x \cdot 1$$

We define this base as e

$$\lim_{h \to 0} \frac{a^h - 1}{h} \overset{def}{=} 1 \qquad a = \underbrace{2.71828\ldots}_{e}$$

Euler's Number

And because of chain rule, we know that if $y = e^{kx}$, then $y' = k \cdot e^{kx}$. So, you can take the derivative of any exponential function by using base e.

Video Example: If $y = 2^x = e^{\ln(2^x)} = e^{x\ln 2}$

$$y' = \frac{d}{dx}\left(e^{x\ln 2}\right) = e^{x\ln 2} \cdot \frac{d}{dx}(x\ln 2)$$

$$= e^{x\ln 2} \ln 2 = 2^x \cdot \ln 2$$

Video Example: If $y = 3^x = e^{\ln(3^x)}$

$$= e^{x\ln 3}$$

$$y' = e^{x\ln 3} \cdot \ln 3 = 3^x \ln 3$$

$$☆\ \frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}b^x = b^x \cdot \ln b$$

$$\frac{d}{dx}e^u = e^u \frac{du}{dx}$$

Practice: Find $f'(x)$ given that:

$$f(x) = \frac{e^{-x^2}}{1 + 5^x}$$

$$f'(x) = \frac{-2xe^{-x^2}(1+5^x) - 5^x \ln 5\, e^{-x^2}}{(1+5^x)^2}$$

$$= -e^{-x^2}\left[\frac{2x(1+5^x) + 5^x \ln 5}{(1+5^x)^2}\right]$$

Practice Problems: 8.2: # Example 4, 4 (skip the one with trig), 5, 6, 7-10, 14

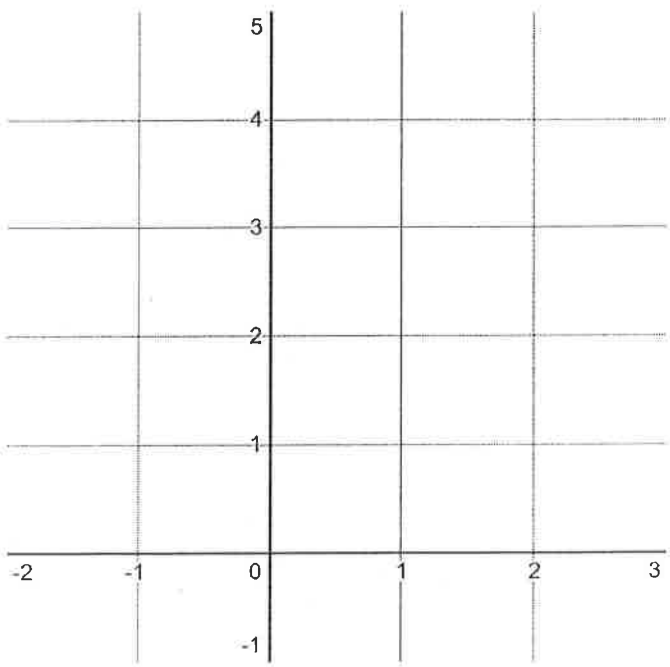
8.2 # 16

8.4: # 4abc

Derivative of the Exponential

Goal:	
• Can take the derivative of e^x with other derivative rules	
• Understands that e is built to be its own derivative.	
Terminology:	
• Euler's Number	
Reminder:	
• Make-up Test on Thursday March 5th after school	

Review: Sketch the function $f(x) = 2^x$ and $g(x) = 3^x$



On the board try to determine $f'(x)$ and $g'(x)$

$$\lim_{h\to 0} \frac{2^{x+h} - 2^x}{h} = \lim_{h\to 0} \frac{2^x 2^h - 2^x}{h} = \lim_{h\to 0} \frac{2^x(2^h - 1)}{h}$$

$$= 2^x \lim_{h\to 0} \frac{2^h - 1}{h}$$

$$= 2^x (0.693\ldots)$$

$$\lim_{h\to 0} \frac{3^{x+h} - 3^x}{h} = \lim_{h\to 0} 3^x\left(\frac{3^h - 1}{h}\right)$$

$$= 3^x (1.096\ldots)$$

From the video we are motivated to find a number $a \in (2.3)$ such that

$$\frac{d}{dx}a^x = a^x \cdot 1$$

$$\boxed{\frac{d}{dx}e^x = e^x}$$

We define this base as e

$$\lim_{h\to 0} a^x\left(\frac{a^h - 1}{h}\right) = a^x \boxed{\lim_{h\to 0}\frac{a^h-1}{h}} \overset{def}{=} 1$$

$a = 2.7182818\ldots\ldots = e$ Euler's Number

And because of chain rule, we know that if $y = e^{kx}$, then $y' = k\, e^{kx}$. So, you can take the derivative of any exponential function by using base e.

Video Example: If $y = 2^x = e^{\ln(2^x)} = e^{x\ln 2}$

$$\frac{d}{dx}\left(e^{x\ln 2}\right) = e^{x\ln 2}\cdot \ln 2$$
$$= 2^x \cdot \ln 2$$

$10^x = y$

$\log_{10} y = x$

$e^x = y$

$\log_e y = x = \ln y$

Video Example: If $y = 3^x =$

$$y = 3^x = e^{\ln(3^x)} = e^{x\ln 3}$$
$$y' = e^{x\ln 3}\cdot \ln 3 = 3^x \ln 3$$

$$\boxed{\begin{aligned}\frac{d}{dx}b^x &= b^x\cdot\ln b\\ \frac{d}{dx}e^u &= e^u\cdot\frac{du}{dx}\end{aligned}}$$

Practice: Find $f'(x)$ given that:

$$f(x) = \frac{e^{-x^2}}{1+5^x}$$

$$f'(x) = \frac{-2xe^{-x^2}(1+5^x) - \ln 5 \cdot 5^x e^{-x^2}}{(1+5^x)^2}$$

$$= \frac{e^{-x^2}\left(-2x + -2x5^x - \ln 5\cdot 5^x\right)}{(1+5^x)^2}$$

Practice Problems: 8.2: # Example 4, 4 (skip the one with trig), 5, 6, 7-10, 14

8.2 # 16

8.4: # 4abc