# Derivative of the Logarithm

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| **Goal:** | • Can take the derivative of $\ln x$ with other derivative rules<br>• Understands how implicit differentiation is useful. |
| **Terminology:** | • Natural Logarithm |
| **Reminder:** | • Make-up Test on Thursday March 5th af~~ter school~~ |

**Review:** Find $\frac{dy}{dx}$ (note that $u$ is not constant)

$$\frac{e^y}{x} = 2^{u^2+u} \Rightarrow e^y = x \cdot 2^{u^2+u}$$

$$e^y \cdot \frac{dy}{dx} = 2^{u^2+u} + x\left[2^{u^2+u} \cdot (2u+1)\left(\frac{du}{dx}\right)\ln 2\right]$$

$$\frac{dy}{dx} = \left(2^{u^2+u} + \ln 2 \cdot x(2u+1)2^{u^2+u}\boxed{\frac{du}{dx}}\right)e^{-y}$$

$$e^y \cdot x^{-1}$$

The natural thing to ask next is what is the derivative of the inverse of the exponential? What is $\frac{dy}{dx}$ if

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$y = \ln x \Rightarrow \boxed{e^y = x}$$

$$\frac{d}{dx}(e^y = x)$$

$$e^y \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

☆ aside

$$\frac{d}{dx} x^n = n x^{n-1}$$

if $n = 0$

$$\frac{d}{dx} x^0 = 0$$

And if you want to find the derivative of a log with any base ...

$$y = \log_b x \iff b^y = x$$

$$\frac{d}{dx}\left(b^y = x\right)$$

$$b^y \cdot \ln b \cdot \frac{dy}{dx} = 1 \Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{b^y \ln b} = \frac{1}{x \ln b}}$$

$$\frac{d}{dx}\ln(u) = \frac{1}{u}\cdot\frac{du}{dx}$$

**Example**: Find $dy/dx$ if we have the following

$$y = \ln(x \cdot \log 2x)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\ln(x \cdot \log 2x)\right)$$

$$= \frac{1}{x\log 2x} \cdot \frac{d}{dx}\left(x \log 2x\right)$$

$$= \frac{1}{x\log 2x} \cdot \left[1 \cdot \log 2x + \cancel{x} \cdot \frac{1}{\cancel{2x}\ln 10} \cdot \frac{d}{dx}(\cancel{2}x)\right]$$

$$\boxed{\frac{\log 2x + \frac{1}{\ln 10}}{x \log 2x}}$$

**Practice**: Find $dy/dx$ if we have the following

$$y = \ln^3\left(\frac{2x-1}{x^2}\right)$$

$$\frac{dy}{dx} = 3\ln^2\left(\frac{2x-1}{x^2}\right) \cdot \frac{x^{\cancel{2}}}{2x-1} \cdot \left[\frac{2x^{\cancel{2}} - 2\cancel{x}(2x-1)}{x^{4\cancel{2}}}\right]$$

$$= 6\ln^2\left(\frac{2x-1}{x^2}\right) \cdot \frac{1}{2x-1} \cdot \left(\frac{\cancel{x} - \cancel{2}x + 1}{x}\right)$$

$$= 6\ln^2\left(\frac{2x-1}{x^2}\right) \frac{1-x}{(2x-1)\,x}$$

**Practice Problems**: 8.4: # 1 (skip the one with trig), 3, 5-10

$2^{u^2+u} \cdot e^{-y}$



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**Review**: Find $\frac{dy}{dx}$ (note that $u$ is not constant)

$$\frac{e^y}{x} = 2^{u^2+u} \Rightarrow e^y = x\, 2^{u^2+u}$$

$$\frac{d}{dx}\left[ e^y = \underbrace{x}\cdot \underbrace{2^{u^2+u}} \right]$$

$$e^y \cdot \frac{dy}{dx} = 2^{u^2+u} + x 2^{u^2+u}(2u+1)\frac{du}{dx}\ln 2$$

$$\frac{dy}{dx} = \frac{2^{u^2+u}}{e^y}\left[1 + x\ln 2\,(2u+1)\frac{du}{dx}\right]$$

The natural thing to ask next is what is the derivative of the inverse of the exponential? What is $\frac{dy}{dx}$ if

$$y = \ln x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

Aside

$$\frac{d}{dx}x^{n} = n x^{n-1}$$

if $n=0$

$$\frac{d}{dx}x^0 = 0\cdot x^{-1} = \frac{0}{x}$$
$$= 0$$

And if you want to find the derivative of a log with any base ...

$$y = \log_b x \iff b^y = x$$

$$\frac{d}{dx}\left(b^y = x\right)$$

$$b^y \cdot \ln b \frac{dy}{dx} = 1$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{b^y \ln b} = \frac{1}{x \ln b}}$$

$$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$$

**Example**: Find $dy/dx$ if we have the following

$$y = \ln(x \cdot \log 2x)$$

$$\frac{dy}{dx} = \frac{1}{x\log 2x} \cdot \frac{d}{dx}\left(x \log 2x\right)$$

$$= \frac{1}{x\log 2x}\left[\log 2x + \frac{1}{\cancel{2x}\ln 10} \cdot \cancel{2} \cdot \cancel{x}\right]$$

$$\frac{dy}{dx} = \frac{1}{x\log 2x}\left[\log 2x + \frac{1}{\ln 10}\right]$$

**Practice**: Find $dy/dx$ if we have the following

$$y = \ln^3\left(\frac{2x-1}{x^2}\right) = \left(\ln\left(\frac{2x-1}{x^2}\right)\right)^3$$

$$\frac{dy}{dx} = 3\ln^2\left(\frac{2x-1}{x^2}\right) \cdot \left(\frac{\cancel{x^2}}{2x-1}\right) \cdot \left[\frac{2x^{\cancel{2}} - 4x^{\cancel{2}} + 2\cancel{x}}{x^{\cancel{4}\,\cancel{2}}}\right]$$

$$= 6\ln^2\left(\frac{2x-1}{x^2}\right)\frac{(1-x)}{(2x-1)x}$$

**Practice Problems**: 8.4: # 1 (skip the one with trig), 3, 5-10