# Derivative Rule Quiz 2

Name: ______________________________ Date: November 4, 2019

| Thinking Strategies | Communication | Modelling & Solving |
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| | | |

1. Use the definition of the derivative to show why the sum/difference rule is true.

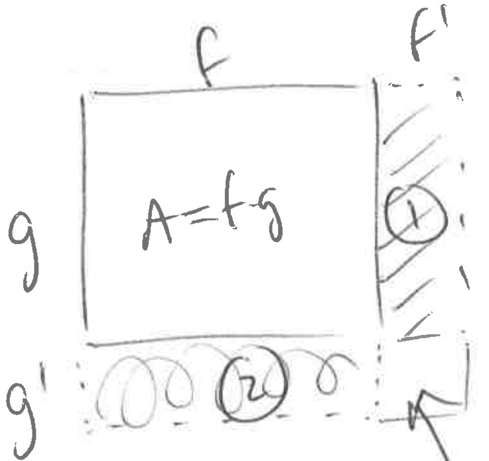
$$\frac{d}{dx}(f \pm g) = \frac{df}{dx} \pm \frac{dg}{dx}$$

$$\lim_{h \to 0} \frac{f(x+h) \pm g(x+h) - f(x) \mp g(x)}{h}$$

$$\lim_{h \to 0} \frac{f(x+h) \pm f(x)}{h} + \lim_{h \to 0} \frac{g(x+h) \pm g(x)}{h}$$

$$f' \pm g' \quad \checkmark$$

2. Illustrate and explain the product rule




change in Area

$A' = (f \cdot g)' = f'g + g'f$

ignore this b/c too small

3. Determine the slope of the tangent line at $x = c$ given that $y = (x^3 - x) \cdot \frac{1}{(x^2+x+1)^2}$

$x^4 + x^2 + 1$

$$y = \frac{x^3 - x}{(x^2+x+1)(x^2+x+1)} \quad f \quad g$$

$x^4 + 2x^3 + 3x^2 + 2x + 1$

$f'(x) = 3x^2 - 1$

$g'(x) = 2(2x+1)(x^2+x+1)$

$$y' = \frac{(3c^2-1)(c^2+c+1)^2 - 2(2c+1)(c^2+c+1)(c^3-c)}{(c^2+c+1)^4}$$

@ $x = c$

4. Determine if $f$ is differentiable at $x = 1$.

$$f(x) = \begin{cases} (\sqrt{x}-1)(x+1) \to 0 & x < 1 \\ \frac{4(x-1)}{x^2-3} \to 0 & x \geq 1 \end{cases}$$

f must be continuous @ $x=1$

on left of 1

$f'(x) = \frac{1}{2}x^{-1/2}(x+1) + (\sqrt{x}-1)$

@ 1 $f'(1) = 1$

on right of 1

$$f'(x) = \frac{4(x^2-3) - 2x(4x-4)}{(x^2-3)^2}$$

@ 1 $f'(1) = \frac{-8}{4} = -2$ not the same

NOT differentiable

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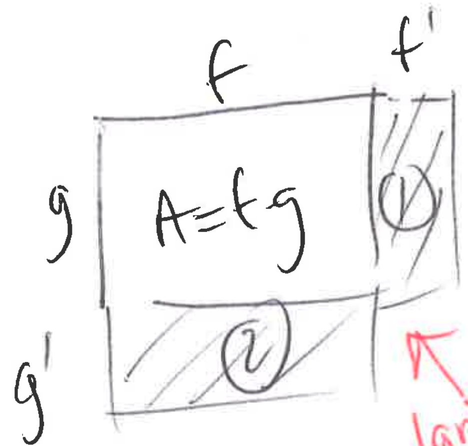
1. Illustrate and explain the product rule.




change in area

$A' = (f \cdot g)' = \underset{1}{f'g} + \underset{2}{g'f}$

Ignore b/c so small

2. Determine the slope of the tangent line at $x = -1$ given that $y = (x^3 + x) \cdot \frac{1}{(x^2 + x + 1)^2}$

$y = \frac{x^3 + x \quad F}{(x^2+x+1)(x^2+x+1) \quad G}$

$F(-1) = -2$

$G(-1) = 1$

$F' = 3x^2 + 1$ $\xrightarrow{@\ x=-1}$ $F'(-1) = 4$

$G' = 2(2x+1)(x^2+x+1) \rightarrow G'(-1) = -2(1) = -2$

$y'(-1) = \frac{F'(-1)G(-1) - G'(-1)F(-1)}{G(-1)^2}$

$= \frac{4(1) - (-2)(-2)}{1^2} = 0$ //

3. Show that the slope of $y = \frac{x}{x^2+1}$ is between $-1$ and $1$.

F

G

$$y' = \frac{1(x^2+1) - (2x)(x)}{(x^2+1)^2}$$

$$= \frac{-x^2+1}{(x^2+1)^2}$$ stays small

$$= \frac{-(x^2-1)}{(x^2+1)^2}$$

$x^2-1 < x^2+1$

numerator < denominator

4. Determine if $f$ is differentiable at $x = 1$.

$$f(x) = \begin{cases} (\sqrt{x}-1)(x+1) \to 0 & x < 1 \\ \frac{4(x-1)}{x^2+3} \to 0 & x \geq 1 \end{cases}$$

needs to be continuous @ $x=1$

on left of 1

$$f'(x) = \frac{1}{2}x^{-1/2}(x+1) + (\sqrt{x}-1)$$

$$f'(1) = 1 + 0 = 1$$

on right of 1

$$f'(x) = \frac{4(x^2+3) - 2x(4x-4)}{(x^2+3)^2}$$

$$f'(1) = \frac{16-0}{16} = 1$$

same

YES differentiable!