But what happens when the degree of the numerator is greater than the degree of the denominator?

Example:

$$\lim_{x\to\infty} \frac{2x^3 - x^2 + 400x}{x^2 + x + 1}$$

Practice:

$$\lim_{x\to\infty} \frac{7x^5 - 2x^4 - 10x^2 + 1}{x^4 + 1}$$

Practice Problems: 5.1: # Anything you feel is valuable (This section is Precalc 12 and early limit review)
5.2: # 1-3 (do what you need), 4, 6, 11
5.6: # 1-3

5.2 # 7-10

Desmos Asymptote Activity

I want you to find the equation to the horizontal and slant asymptotes by using Desmos to graph and compare the rational function to the equation to the asymptote as I showed in class.

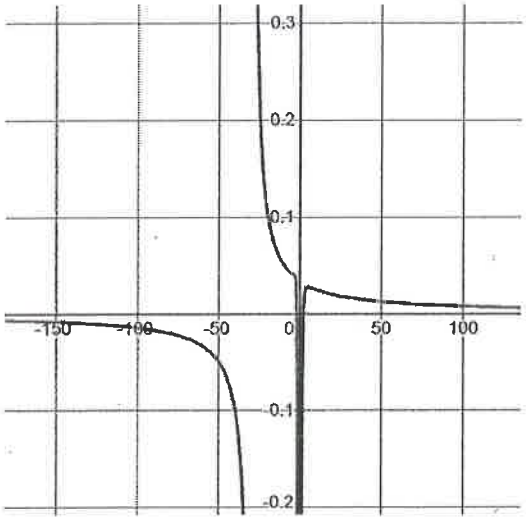
Go to:

desmos.com/calculator/rhnw0r4upz

Find the equation to the horizontal/slant asymptote and graph the asymptote along with the graph.

1.

$$\frac{x^6 - 5x^3 - 100}{x^7 + 30x^6}$$ - two

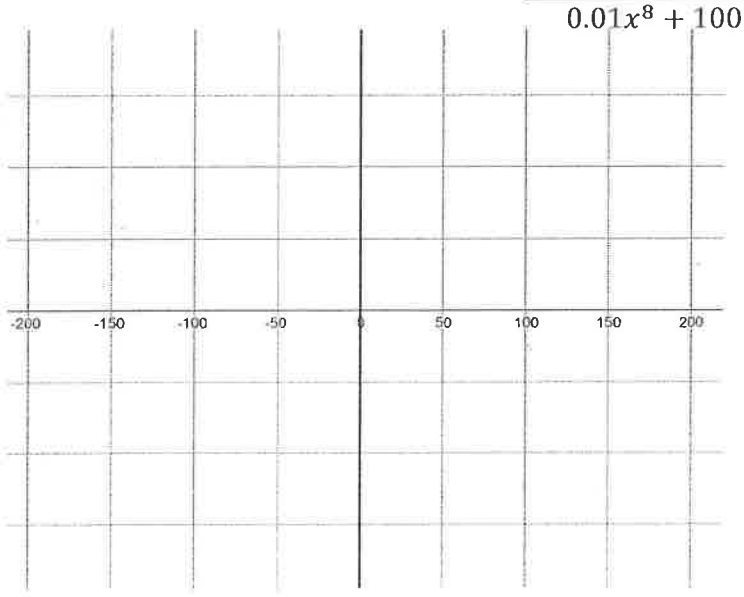


$$\lim_{x\to\infty} \frac{x^6 - 5x^3 - 100}{x^7 + 30x^6}$$

$$= \lim_{x\to\infty} \frac{1}{x^7} = 0$$

2.

$$\frac{-x^5 + 100x^3 - 200}{0.01x^8 + 100}$$

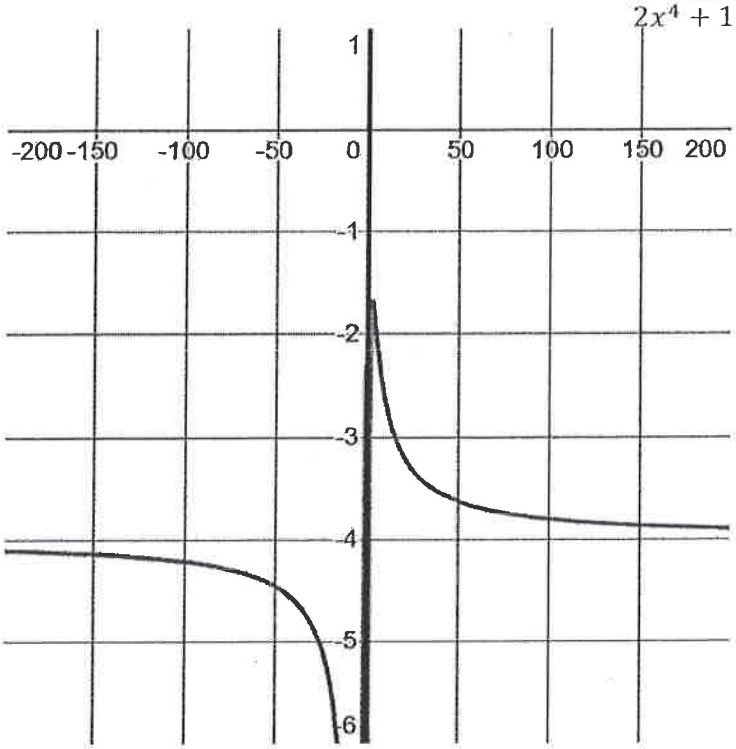


$$\lim_{x\to\infty} \frac{-x^5 + 100x^3 - 200}{0.01x^8 + 100}$$

$$= \lim_{x\to\infty} \frac{1}{0.01x^8} = 0$$

3.

$$\frac{-8x^4 + x^3 - 20}{2x^4 + 10x^3 - 18x}$$



$\lim_{x \to \infty} \frac{-8x^4 + x^3 - 20}{2x^4 + 10x^3 - 18x}$

less than x^4 (1)

same

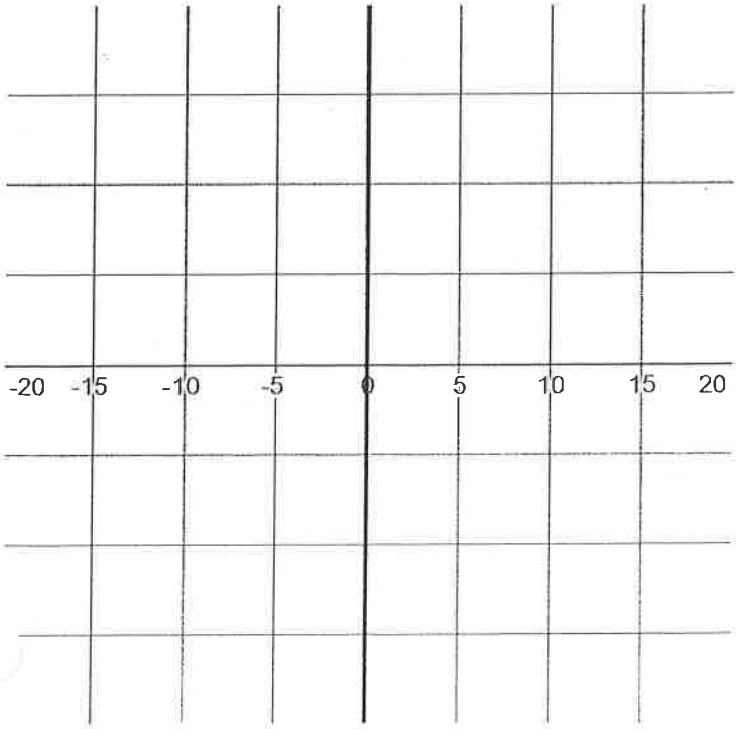
less than x^4 on top (2)

$= \lim_{x \to \infty} \frac{-8x^4}{2x^4}$

$= -4$

4.

$$\frac{20x^3 - 6x^2 + 15x}{4x^3 - 10x^2 + 1}$$



$\lim_{x \to \infty} \frac{20x^3 - 6x^2 + 15x}{4x^3 - 10x^2 + 1}$

less than bottom x^3 (1)

same

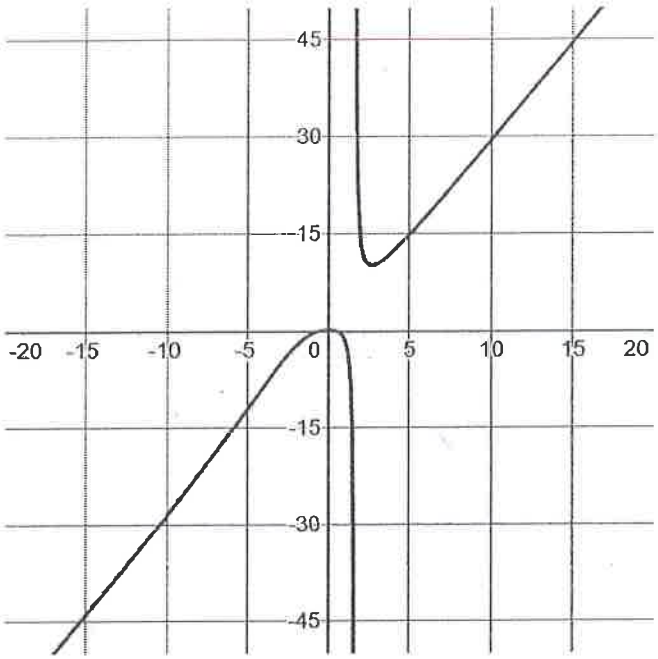
less than x^3 on top (2)

$= \lim_{x \to \infty} \frac{20x^3}{4x^3}$

$= 5$

5.

$$\frac{3x^4 + 10x^2 - 3}{x^3 + 7x - 16}$$



less than x^3 bottom (1)

$$\lim_{x\to\infty} \frac{3x^4 + 10x^2 - 3}{x^3 + 7x - 16}$$

less than x^2 (2)

1 bigger (need 1 more term x^2)

$+6x^2$

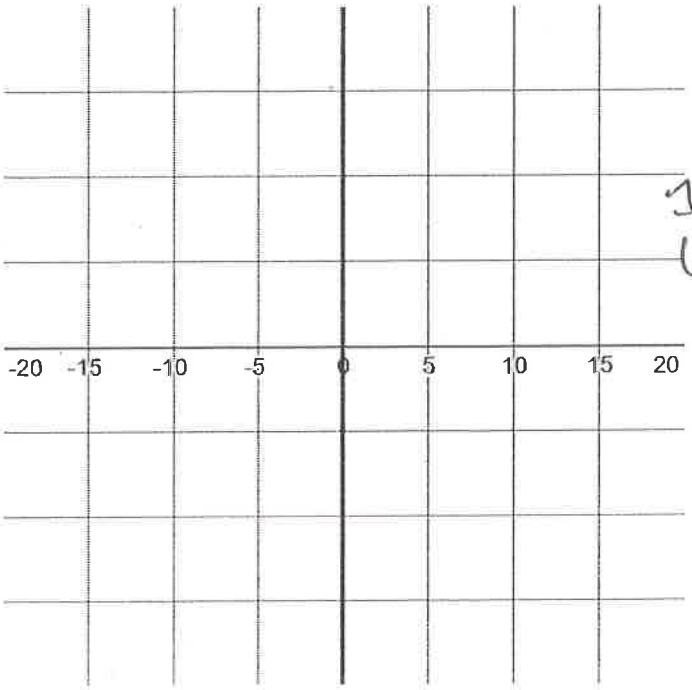
$$\equiv \lim_{x\to\infty} \frac{3x^4}{x^3} = \lim_{x\to\infty} 3x$$

Slant asymptote

$y = 3x$

6.

$$\frac{-x^5 - 4x^3 + 20x}{2x^4 + 5x^2 - 10}$$



less than x^4 on bottom (1)

$$\lim_{x\to\infty} \frac{-x^5 - 4x^3 + 20x}{2x^4 + 5x^2 - 10}$$

less than x^3 (2)

1 bigger (need 1 more term, x^3)

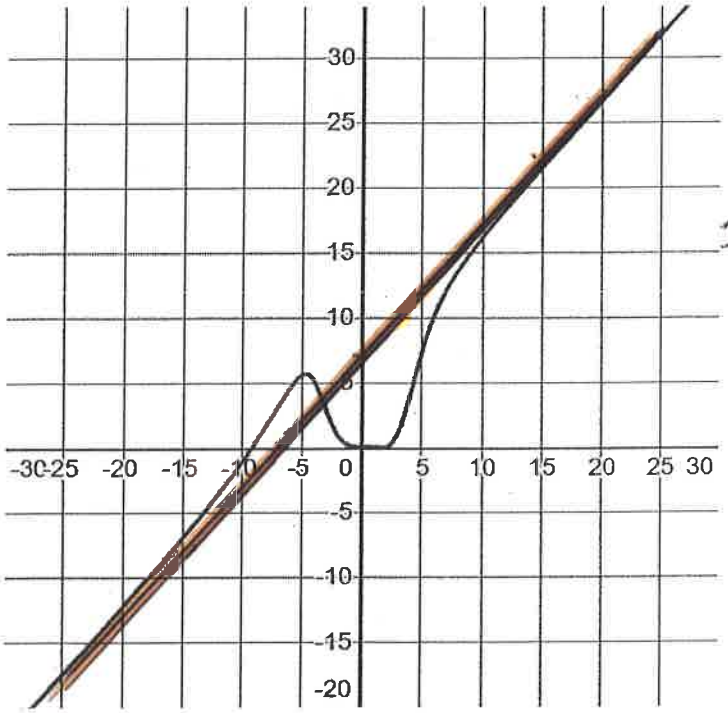
$$\equiv \lim_{x\to\infty} \frac{-x^5}{2x^4} = \lim_{x\to\infty} \frac{-x}{2}$$

slant asymptote

$y = -\frac{x}{2}$

7.

$$\frac{x^5 + 7x^4 - 20x^3 + 30}{x^4 - 10x^2 + 300}$$



less than x^4 on bottom (1)

$$\lim_{x\to\infty} \frac{x^5+7x^4-20x^3+30}{x^4-10x^2+300}$$

less than x^3 (2)

1 more on top so need 1 more term (x^3)

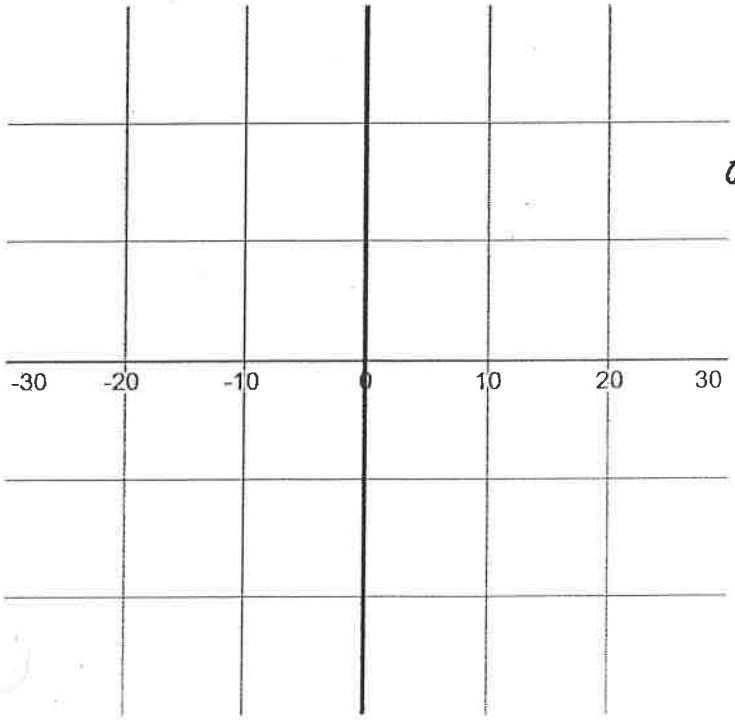
$$\equiv \lim_{x\to\infty} \frac{x^5+7x^4}{x^4} = \lim_{x\to\infty}(x+7)$$

Slant asymptote

$y = x + 7$

8.

$$\frac{2x^4 - 10x^3 + 30x^2 - 100}{x^3 + 29x}$$



less than x^3 on bottom (1)

$$\lim_{x\to\infty} \frac{2x^4-10x^3+30x^2-100}{x^3+29x}$$

1 more on top (need x^2 term)

less than x^2 (2)

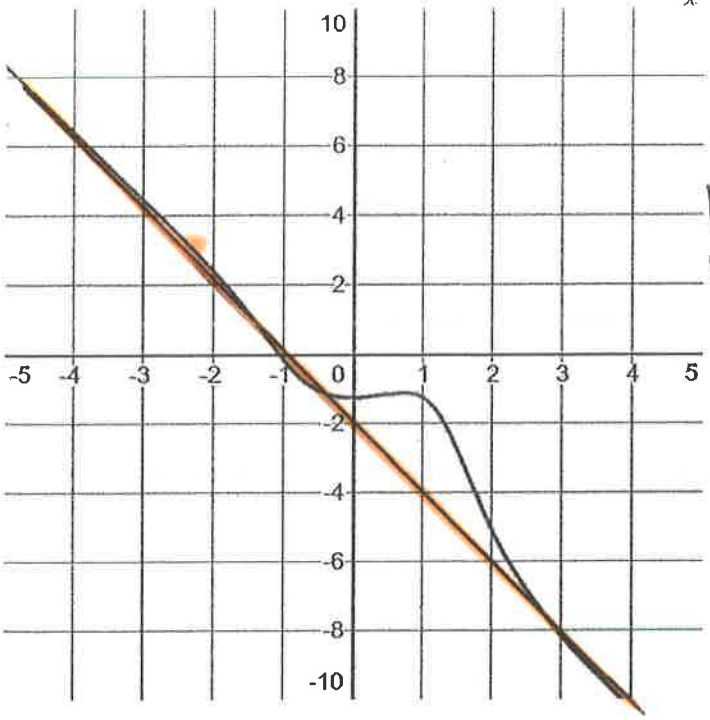
$$\equiv \lim_{x\to\infty} \frac{2x^4-10x^3}{x^3} = \lim_{x\to\infty}(2x-10)$$

Slant asymptote 2

$y = 2x - 10$

9.

$$\frac{2x^5 - 3x^2 + 5}{-x^4 + x^3 - 4}$$



less than x^4 ①

$$\lim_{x\to\infty} \frac{2x^5 - 3x^2 + 5}{-x^4 + x^3 - 4}$$ less than x^3 ②

1 more need x^3

$$= \lim_{x\to\infty} \frac{2x^5}{-x^4 + x^3} = \lim_{x\to\infty} \left(-2x - 2 + \frac{2x^3}{-x^4 + x^3}\right)$$

$$\frac{-2x - 2}{-x^4 + x^3 \,\big)\, 2x^5}$$

$$-(-2x^4 - 2x^4)$$

$$2x^4$$

$$+2x^4 - 2x^3$$

$2x^3$ → remainder

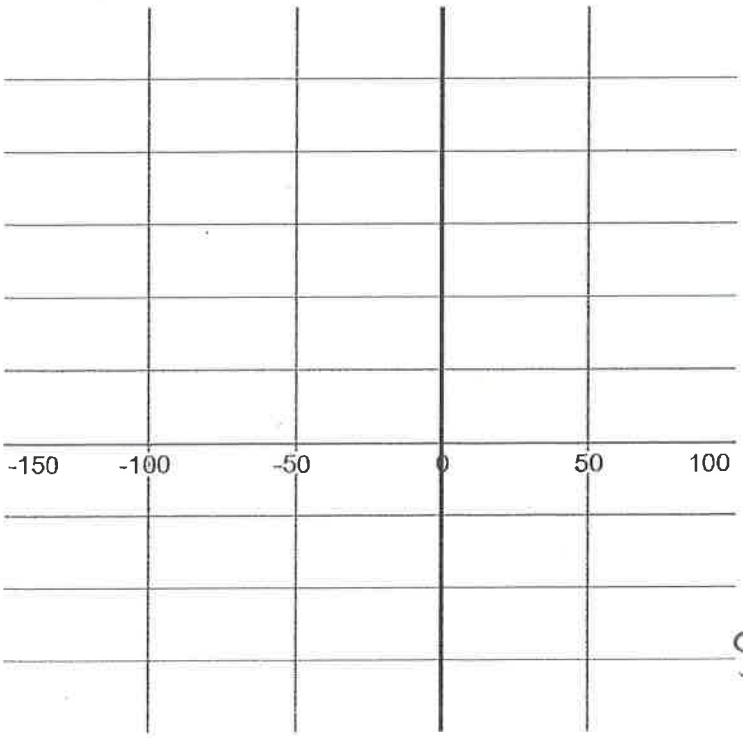
$\frac{2x^3}{-x^4 + x^3} \to 0$

slant asym.

$$\boxed{y = -2x - 2}$$

10.

$$\frac{-x^3 + 3x - 10}{3x^2 + 18x - 20}$$



less than x^2 ①

$$\lim_{x\to\infty} \frac{-x^3 + 3x - 10}{3x^2 + 18x - 20}$$ less than x ②

1 more need x

$$= \lim_{x\to\infty} \frac{-x^3}{3x^2 + 18x} = \lim_{x\to\infty} \left(-\frac{1}{3}x + 2 + 0\right)$$

$$-\frac{1}{3}x + 2$$

$$3x^2 + 18x \,\big)\, -x^3$$

$$-(-x^3 - 6x^2)$$

$$6x^2$$

$6x^2$ + Renca.

Slant asym.

$$\boxed{y = -\frac{1}{3}x + 2}$$

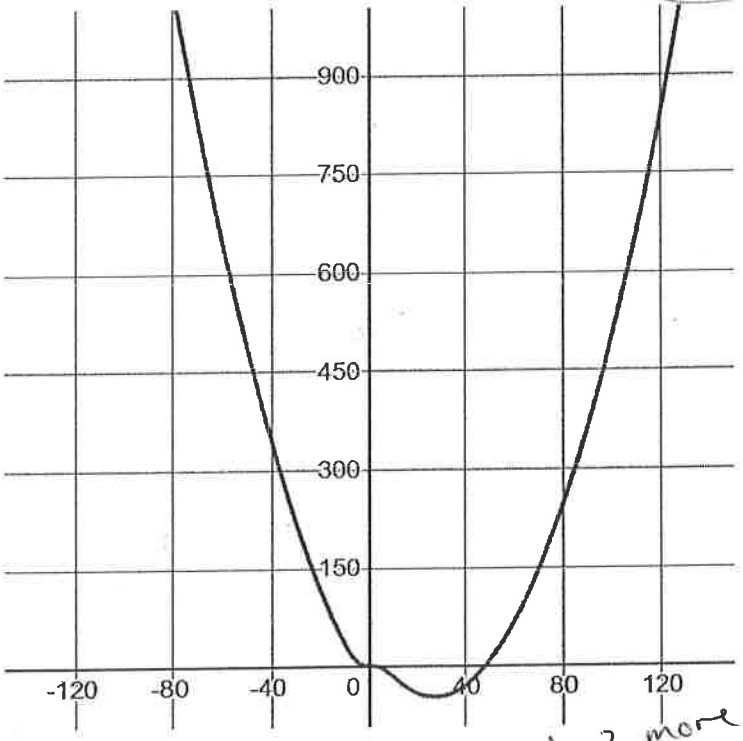
11.

need 2 more x and x^0

less than x^2 (1)

$$\frac{0.1x^4 - 5x^3 + 8x^2 - 10x + 20}{x^2 + 100}$$ all

$\lim_{x \to \infty}$



$$x^2+100 \overline{)\ 0.1x^4 - 5x^3 + 8x^2}$$

$0.1x^2 - 5x - 2$

$-(0.1x^4 + 10x^2)$

$-5x^3 - 2x^2$

$(-5x^3 - 500x)$

$-2x^2 + 500x$

$-2x^2 - 200$

Remainder

asymptote is

$$y = 0.1x^2 - 5x - 2$$

12.

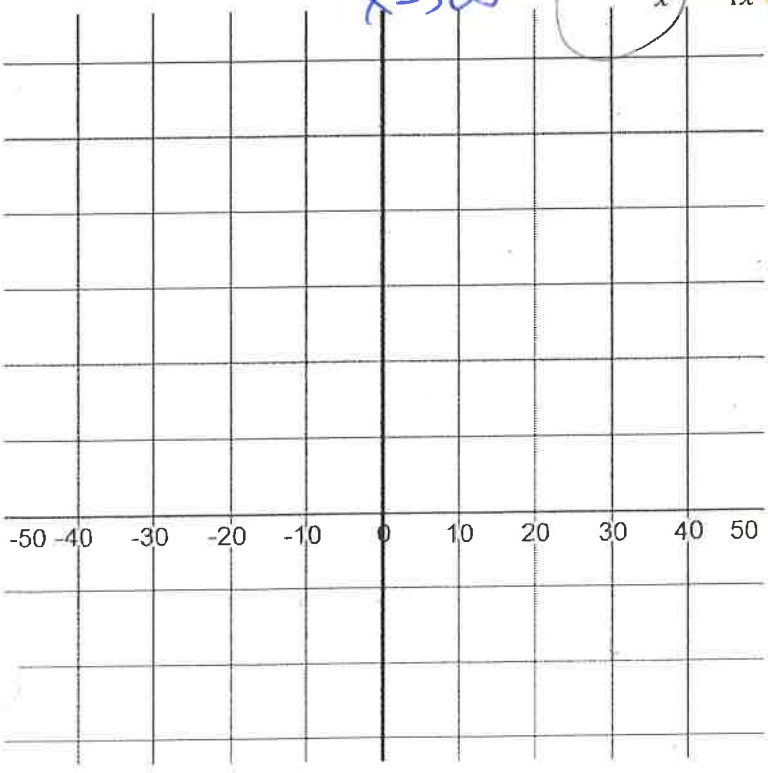
need x^2 and x more

less than x^3 (1)

$$\frac{(x^5 - 2x^4 + 5x^3 - 43x^2 + 30)}{x^3 - 4x + 100}$$

less than x

$\lim_{x \to \infty}$



$$x^3-4x \overline{)\ x^5 - 2x^4 + 5x^3}$$

$x^2 - 2x + 9$

$-(x^5 - 4x^3)$

$-2x^4 + 9x^3$

$-(-2x^4 + 8x^2)$

$9x^3 - 8x^2$

$9x^3 - \cdots$

Remainder

asymptote is

$$y = x^2 - 2x + 9$$

In general, what strategies are you using and what patterns are you noticing? Be as specific as possible.