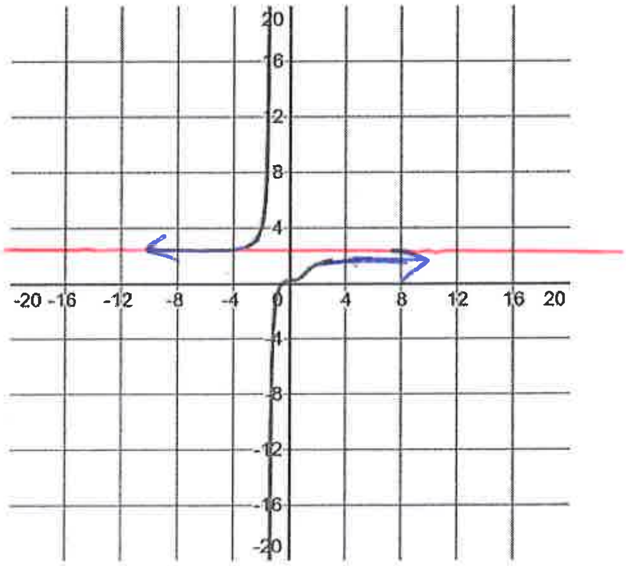
More Slant and Horizontal Asymptote Practice

Find the equation to the slant/horizontal asymptote using limits as $x \to \infty$. Complete the graphs.

Reminder: Quiz on Tuesday: Slant/Horizontal asymptotes and Concavity (Wednesday's Lesson)

1.

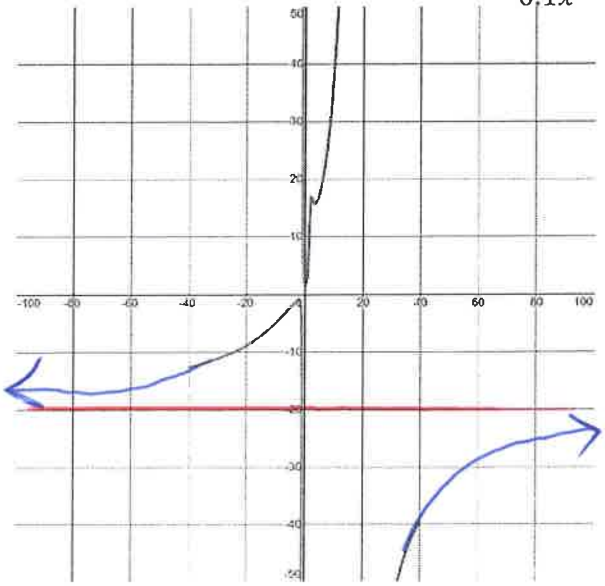
$$\frac{4x^3 - 2x^2 + 1}{2x^3 - x + 4}$$



$$\lim_{x \to \infty} \frac{4x^3}{2x^3} = 2$$

2.

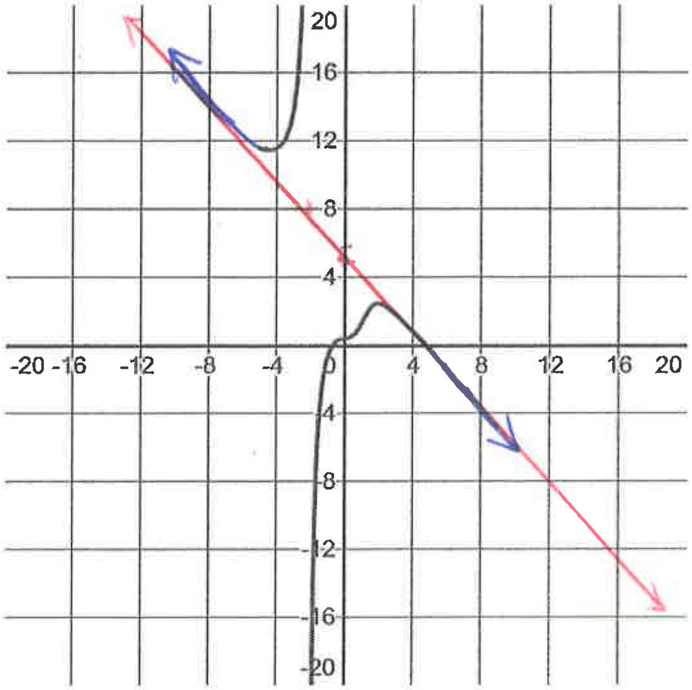
$$\frac{-2x^4 - 3x^3 + 5x - 10}{0.1x^4 - 2x^3 + 4x^2 - 5}$$



$$\lim_{x \to \infty} \frac{-2x^4}{0.1x^4} = -20$$

3.

$$\frac{x^4 - 5x^3 + x^2 - 2}{-x^3 + 2x - 5}$$

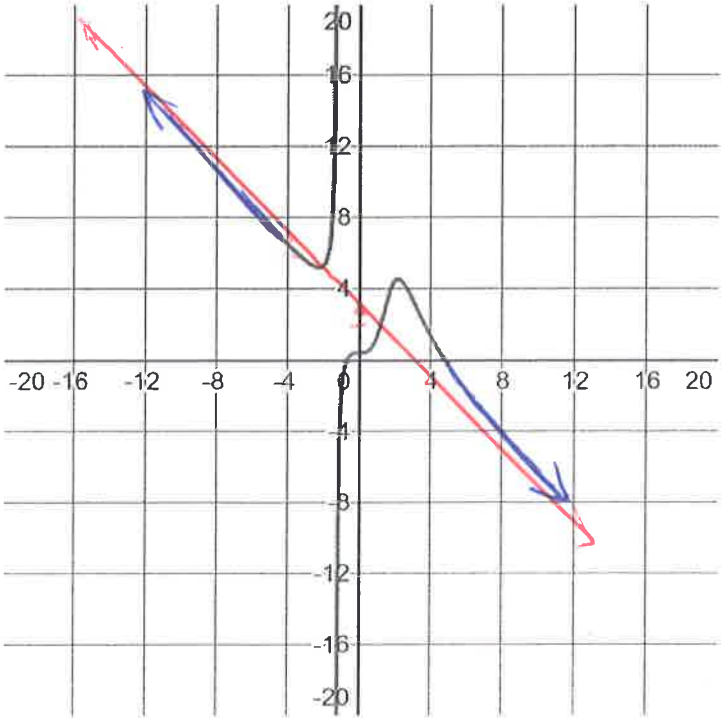


$$= \lim_{x \to \infty} \frac{x^4 - 5x^3}{-x^3}$$

$$= -x + 5$$

4.

$$\frac{x^4 - 5x^3 + x^2 - 2}{-x^3 + 2x^2 - 5}$$



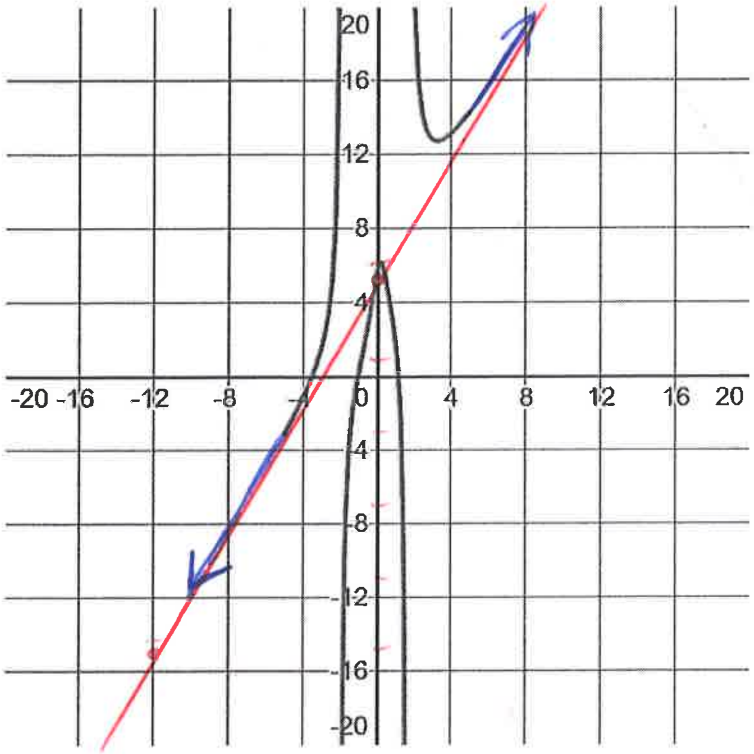
$$= \lim_{x \to \infty} \frac{x^4 - 5x^3}{-x^3 + 2x^2}$$

$$\begin{array}{rl} & -x + 3 \\ -x^3 + 2x^2 & \overline{\big) \, x^4 - 5x^3} \\ & -(x^4 - 2x^3) \\ \hline & -3x^3 \end{array}$$

$$y = -x + 3$$

5.

$$\frac{5x^5 + 15x^4 - 10x^3 - 23}{3x^4 - 9x^2 + 3x - 4}$$

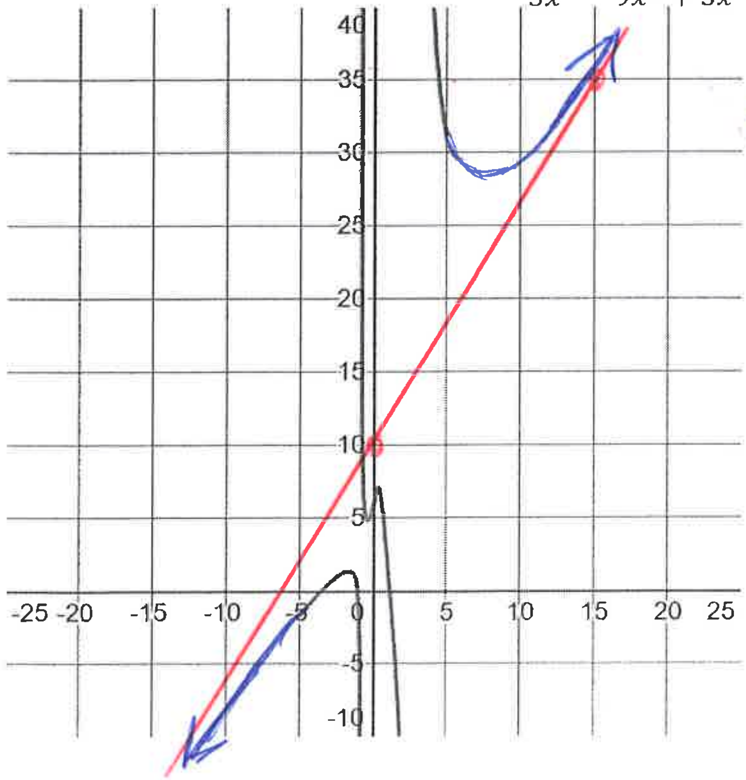


$$= \lim_{x\to\infty} \frac{5x^5 + 15x^4}{3x^4}$$

$$= \frac{5}{3}x + 5$$

6.

$$\frac{5x^5 + 15x^4 - 10x^3 - 23}{3x^4 - 9x^3 + 3x - 4}$$



$$= \lim_{x\to\infty} \frac{5x^5 + 15x^4}{3x^4 - 9x^3}$$

$$\frac{5}{3}x + 10$$

$$3x^4 - 9x^3 \overline{)\,5x^5 + 15x^4}$$

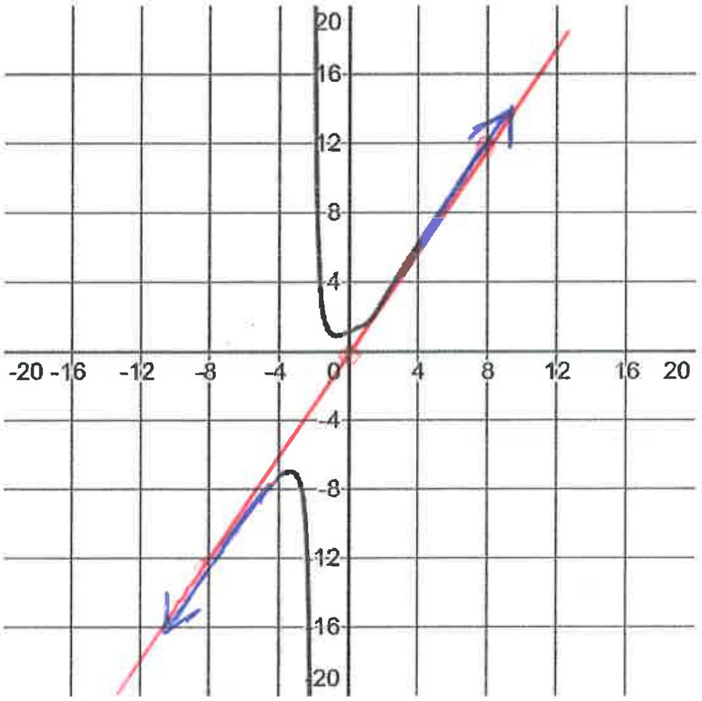
$$-(5x^5 - 15x^4)$$

$$30x^4$$

$$y = \frac{5}{3}x + 10$$

7.

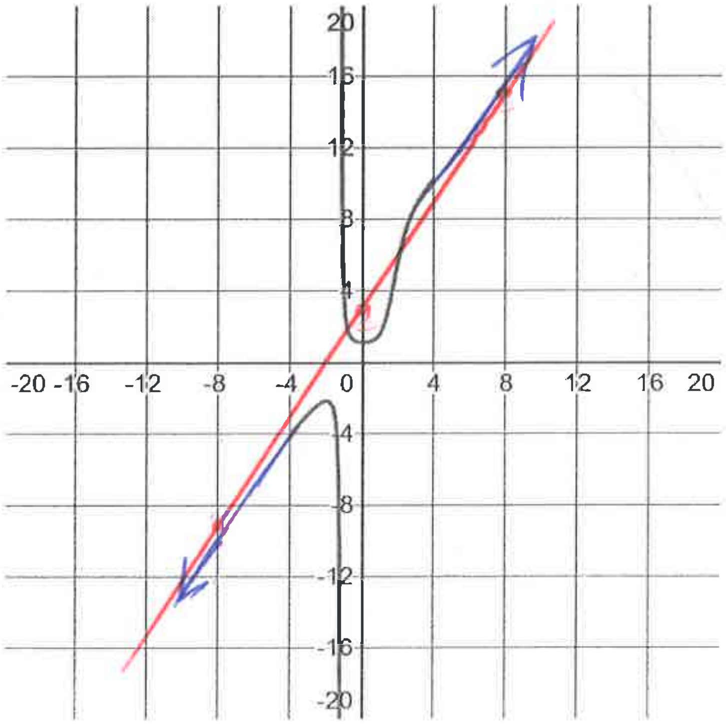
$$\frac{3x^4 - 2x^2 + 10}{2x^3 - 4x + 9}$$



$$= \lim_{x\to\infty} \frac{3x^4}{2x^3} = \frac{3x}{2}$$

8.

$$\frac{3x^4 - 2x^2 + 10}{2x^3 - 4x^2 + 9}$$



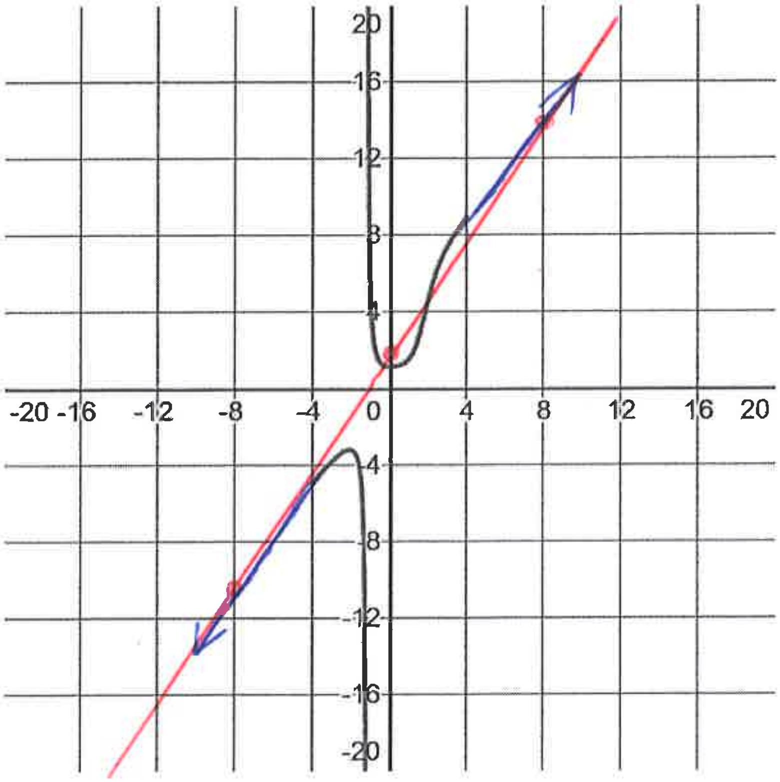
$$= \lim_{x\to\infty} \frac{3x^4 \cancel{2}}{2x^3 - 4x^2}$$

$$\begin{array}{r}
\frac{3}{2}x + 3 \\
2x^3 - 4x^2 \overline{\big) \; 3x^4 \qquad\qquad} \\
-(3x^4 - 6x^3) \\
\hline
6x^3 \\
\vdots
\end{array}$$

$$y = \frac{3}{2}x + 3$$

9.

$$\frac{3x^4 - 2x^3 + 10}{2x^3 - 4x^2 + 9}$$



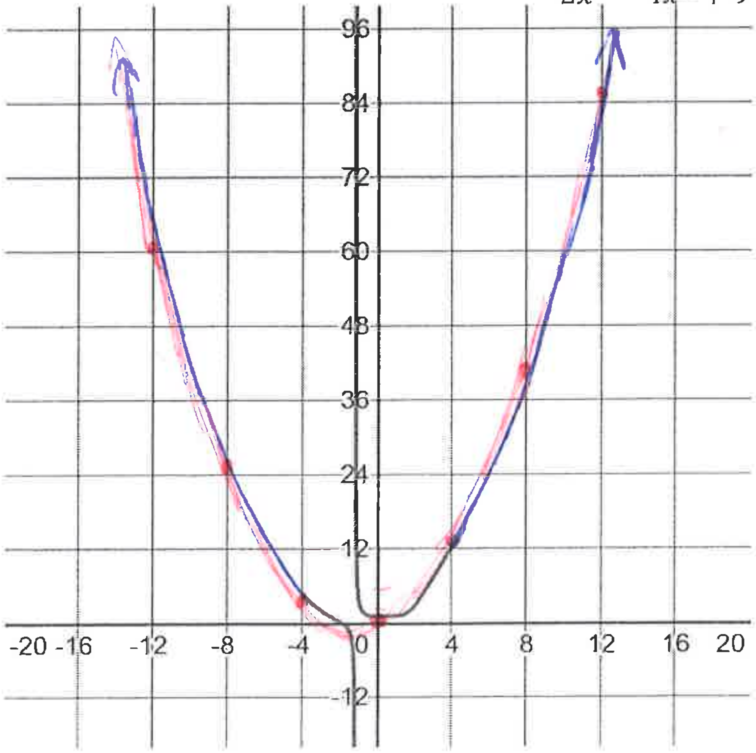
$$= \lim_{x\to\infty} \frac{3x^4 - 2x^3}{2x^3 - 4x^2}$$

$$\begin{array}{r} \frac{3}{2}x + 2 \\ 2x^3 - 4x^2 \overline{\big) 3x^4 - 2x^3} \\ -(3x^4 - 6x^3) \\ \hline 4x^3 \\ 4x^3 + \cdots \end{array}$$

$$y = \frac{3}{2}x + 2$$

10.

$$\frac{x^5 - 2x^3 + 10}{2x^3 - 4x^2 + 9}$$



$$= \lim_{x\to\infty} \frac{x^5 - 2x^3}{2x^3 - 4x^2}$$

need x term

$$\begin{array}{r} \frac{1}{2}x^2 + x + 1 \\ 2x^3 - 4x^2 \overline{\big) x^5 - 2x^3} \\ -(x^5 - 2x^4) \\ \hline 2x^4 - 2x^3 \\ -(2x^4 - 4x^3) \\ \hline 2x^3 \end{array}$$

$$y = \frac{1}{2}x^2 + x + 1$$

11. Consider f given below. Determine $f(6 \times 10^{23})$ to the nearest whole number.

$$f(x) = \frac{3x^5 - 12x^3 + 13}{6x^4 - 48x^3 + 20}$$

for large x $f(x) \approx \dfrac{3x^5}{6x^4-48x^3} = \frac{1}{2}x + 4 + \overset{0}{\cancel{\text{small}}}$

$$\begin{array}{r|l} & \frac{1}{2}x + 4 \\ \hline 6x^4-48x^3 & 3x^5 \\ & -(3x^5-24x^4) \\ \hline & 24x^4 \\ & 24x^4 + \dots \end{array}$$

23 zeros ≡ 24 digits

$f(6\times10^{23}) = \frac{1}{2}(6\times10^{23}) + 4 = 3\times10^{23} + 4$

$= \underline{300}\ \underline{000}\ \underline{000}\ \underline{000}\ \underline{000}\ \underline{000}\ \underline{000}\ \underline{004}$

= ~~trill~~ ~~billio~~ ~~million~~

~~thousand~~ ~~trillion~~

~~three hundred~~

four

Solutions:

1. $y = 2$
2. $y = -20$
3. $y = -x + 5$
4. $y = -x + 3$
5. $y = \frac{5}{3}x + 5$
6. $y = \frac{5}{3}x + 10$
7. $y = \frac{3}{2}x$
8. $y = \frac{3}{2}x + 3$
9. $y = \frac{3}{2}x + 2$
10. $y = \frac{1}{2}x^2 + x + 1$
11. $3 \times 10^{23} + 4$ or 300 000 000 000 000 000 000 004