First Derivative Test and Newton's Method Practice

1. Find all maximum and minimums for the following functions:

a. $f(x) = x^3 - 3x^2 + 5$

$f'(x) = 3x^2 - 6x$

$= 3x(x-2)$

$x = 0, 2$

x		0		2	
$f'(x)$	$+$	0	$-$	0	$+$
	$\nearrow$		$\searrow$		$\nearrow$

max @ $x=0 \to f(0)=5$

min @ $x=2 \to f(2)=1$

b. $g(x) = -2x^3 + 24x$ on $[-1, 3]$

$g'(x) = -6x^2 + 24$

$= -6(x^2 - 4)$

$x = \pm 2$

x		-2	[		2		]
$g'(x)$	$-$	0		$+$	0	$-$	
	$\searrow$			$\nearrow$	max	$\searrow$	
		min					

min @ $x = -1 \to g(-1) = -22$ absolute

max @ $x = 2 \to g(2) = 32$ absolute

min @ $x = 3 \to g(3) = 18$ local

c. $h(x) = \frac{1}{2}x^4 + 8x - 5$

$h'(x) = 2x^3 + 8$

$x = -\sqrt[3]{4}$

x		$-\sqrt[3]{4}$	
$h'(x)$	$-$	0	$+$
	$\searrow$	min	$\nearrow$

absolute min @ $x = -\sqrt[3]{4}$

$\to h(-\sqrt[3]{4}) = \cancel{10.87}\ldots$

$= -14.524\ldots$

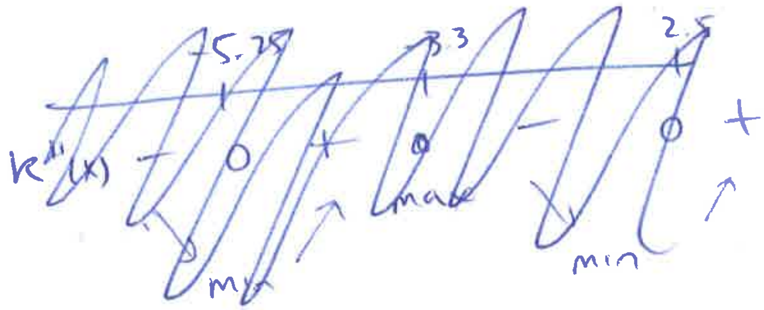
2. Use Newton's method to find the zeros to the slope so you can find the local extremas. (Note: This function was made so the slope has zeros at very recognizable fractions in the interval $[-7, 4]$)

$$k(x) = \frac{1}{1000}(14.4x^5 + 169.5x^4 + 391.667x^3 - 2050x^2 - 10500x)$$

$k'(x) = 72x^4 + 678x^3 + 1175.01x^2 - 4100x - 10500$

$k''(x) = 288x^3 + 2034x^2 + 2350.02x - 4100$

$$x_{root} = \frac{-k''(x)}{k'(x)} + x$$



$z_0 = -7 \rightarrow x = -5.25$

$z_0 = -5 \rightarrow x = -5.25$

$z_0 = -4 \rightarrow x = -3.33$

$z_0 = -2 \rightarrow x = -3.33$

$z_0 = 0 \rightarrow$

$z_0 = 2 \rightarrow x = 2.5$

		-5.25		-3.3		2.5	
$k'(x)$	+	0	−	0	−	0	+
	↗	max	↘		↘	min	↗

~~min @ $x = -7 \rightarrow k(-7) =$~~

max @ $x = -5.25 \rightarrow k(-5.25) = 13.281$

min @ $x = 2.5 \rightarrow k(2.5) = -24.915$

~~max @ $x = 4 \rightarrow k(4) =$~~

3. Use Newton's method to find the zeros of the slope so you can find ALL extremas of the function on the interval $[-2, 1]$ (This function was made so the coefficients look pretty and that was it)

$$L(x) = x^5 + 2x^4 - 3x^3 - 4x^2 + 5x$$

$L'(x) = 5x^4 + 8x^3 - 9x^2 - 8x + 5$

$L''(x) = 20x^3 + 24x^2 - 18x - 8$

$z_0 = 0 \rightarrow x = 0.50742\ldots = \alpha$

$z_0 = -5 \rightarrow x = -1.97074\ldots = \beta$

$z_0 = 5 \rightarrow x = 0.93399\ldots = \gamma$

$z_0 = -1 \rightarrow x = -1.07066\ldots = \delta$

		$[$	β		δ		α		γ	$]$	
$L'(x)$	$+$		0	$-$	0	$+$	0	$-$	0		$+$
	$\nearrow$		max	$\searrow$	min	$\nearrow$	max	$\searrow$	min		$\nearrow$

local min @ $x = -2$

max @ $x = \beta$

min @ $x = \delta$ → back for values

max @ $x = \alpha$

min @ $x = \gamma$

local max @ $x = 1$

4. Why does Newton's method fail to find zeros of the following function?

$$M(x) = x^4 + x + 0.5$$

$M'(x) = 4x^3 + 1$

$M'(x) = 0$ @ $x = -\sqrt[3]{1/4}$

$-\sqrt[3]{4^{-1}}$

$M'(x)$ $-$ 0 $+$

$\searrow$ min $\nearrow$

absolute min @ $x = -\sqrt[3]{1/4}$

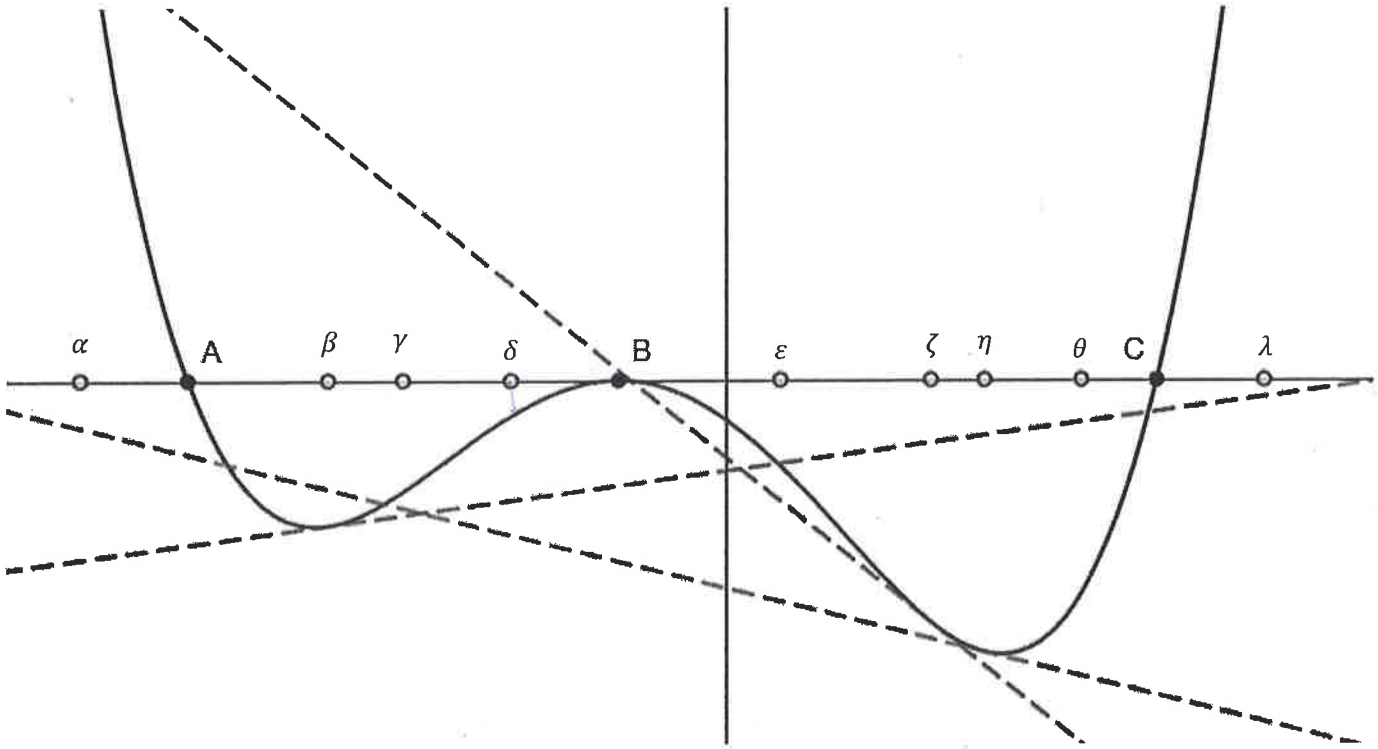
$M(-\sqrt[3]{1/4}) = 0.027 > 0$

smallest value of M

so $M(x) > 0$ for every x

no zeros! Newtons method will bounce around $x = -0.62996\ldots$ as it looks like a zero but then shoot away.

5. What zeros will the different choices of z_0 find using Newton's method? (the circled points labeled in Greek letters). Some tangent lines are given.



What would happen if $z_0 = \delta$ but the entire graph was shifted down very slightly so that there was no zero at B?

If $z_0 = \delta$ but the graph moves down then we would start to go to B but we would hit a slope ~ 0 and be shot off somewhere not near B. Who knows where it will go but not a good pick.

Solutions:

1.
   a. Local Max: $f(0) = 5$; Local Min: $g(2) = 1$
   b. Absolute Max: $g(2) = 32$; Absolute Min: $g(-1) = -22$; Local Min: $g(3) = 18$ (endpoint)
   c. Absolute Min: $h(-\sqrt[3]{4}) = -14.524$; No maximums
2. Local Max: $k(-5.25) = 13.281$; Local Min: $k(2.5) = -24.915$; Note that $x = -3.33$ is not a critical point since $k'(x)$ does not change signs around it
3. Absolute Max: $L(0.507) = 1.281$; Absolute Min: $L(-1.071) = -5.035$; Local Min: $L(0.934) = 0.969$; Local Min: $L(-2) = -2$ (endpoint); Local Max: $L(-1.971) = -1.986$; Local Max: $L(1) = 1$ (endpoint)
4. Hint: Think about the extremas of the curve. Another hint is in the next problem.
5. A is found by: α and η

   B is found by: $\gamma, \delta, \varepsilon$, and ζ

   C is found by: β, θ, and λ

   If the graph is shifted down then using $z_0 = \delta$ will result in what happens with the previous question.