

First Derivative Test for Extrema

Goal:

- Understands under what conditions extreme values occur.
- Understands that the first derivative sign tells us when the function is increasing or decreasing.
- Remembers to check endpoints for absolute extremas.

Terminology:

- Extrema
- Local and absolute
- Critical Point
- First Derivative Test

Quiz on Monday Test Feb 4

Review: Find the zeros to the curve, $f(x) = x^4 - 2x - 3$

$f'(x) = 4x^3 - 2$ when does

$$x = \frac{-f'(z_0)}{f''(z_0)} + z_0$$

$$= \frac{-(4x^3 - 2)}{12x^2} + x$$

$x = -1$ when $z_0 \leq 0.5$ $x = 1.5747...$ when $z_0 > 0.5$

$f'(x) = 0$

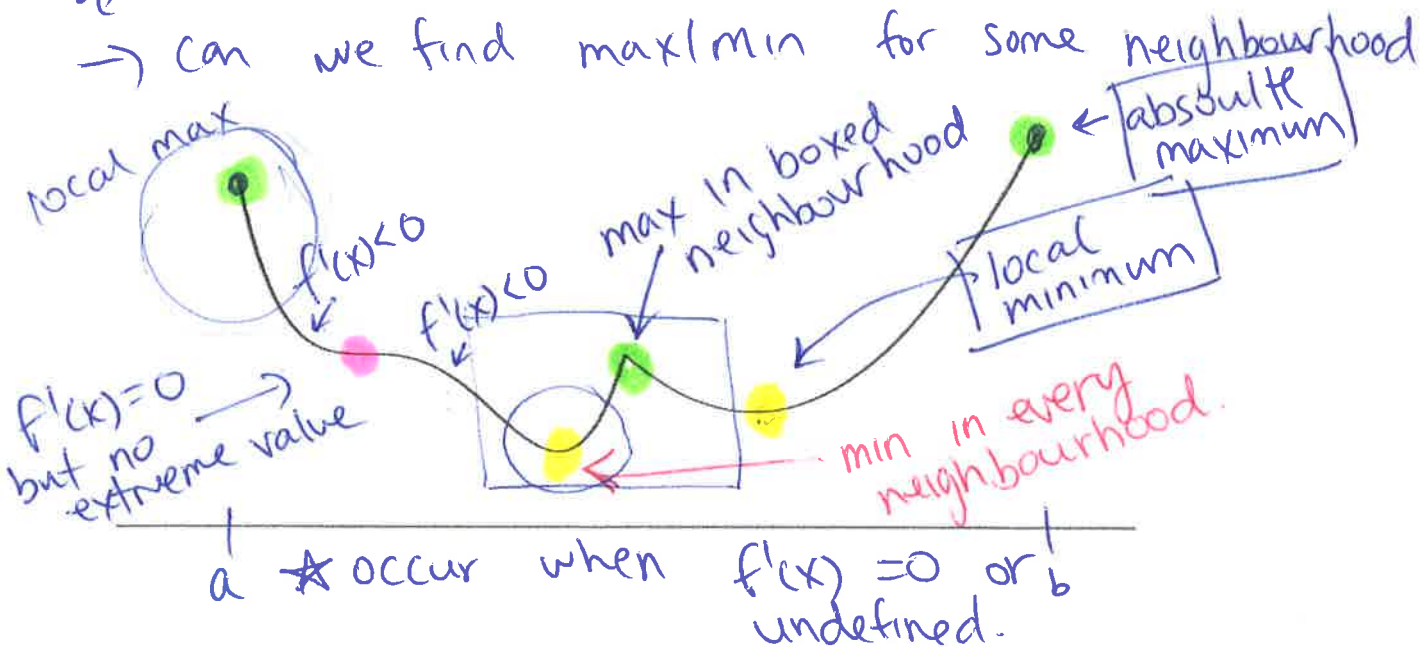
$4x^3 = 2$

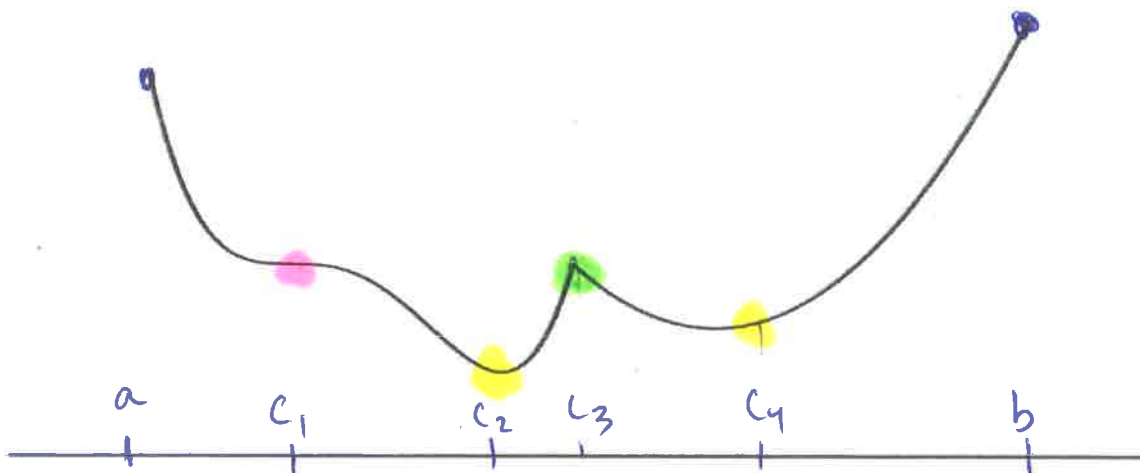
$x = 0.79$

avoid for z_0

What are the conditions for the function to have a maximum or minimum value?

- polynomial even degree? will give us 1
- largest or smallest
- can we find max/min for some neighbourhood





x	a	c_1	c_2	c_3	c_4	b	
$f'(x)$?	-	0	-	0	+	?

Behaviour of f

decreas. decreas. incr. dec. incr.

Shape



Extrema

minimum max min

This is the First Derivative Test:

- Local MAXIMUMs occur when
- Local minimums occur when

$f'(x)$ moves from positive to negative
 $f'(x)$ moves from negative to positive

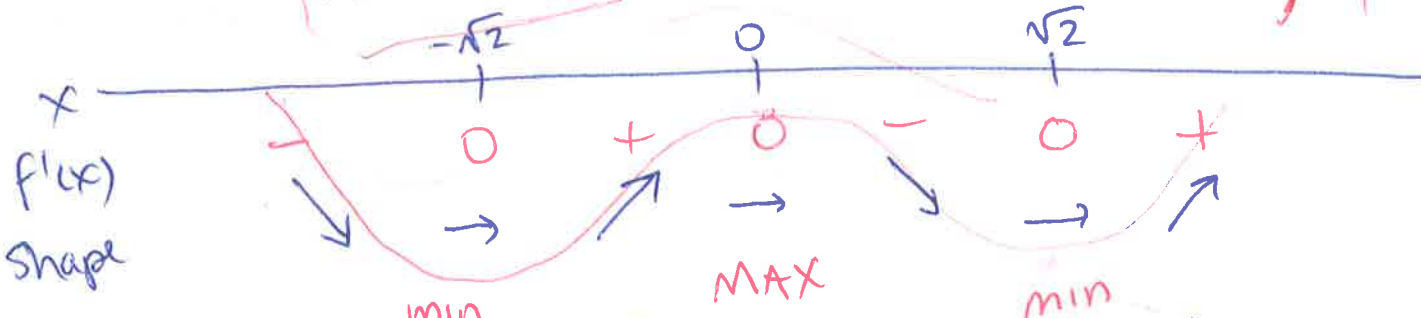
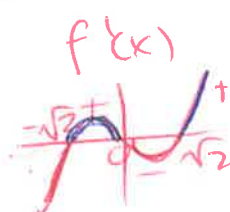
Example: Find the local extrema of the function

$$f(x) = x^4 - 4x^2 - 2$$

for $x \in \mathbb{R}$

$$f'(x) = 4x^3 - 8x = 0$$

$$4x(x^2 - 2) = 0 \Rightarrow x = 0, \pm\sqrt{2}$$



local min @ $x = \pm\sqrt{2}$, $f(-\sqrt{2}) = -8 = f(\sqrt{2})$

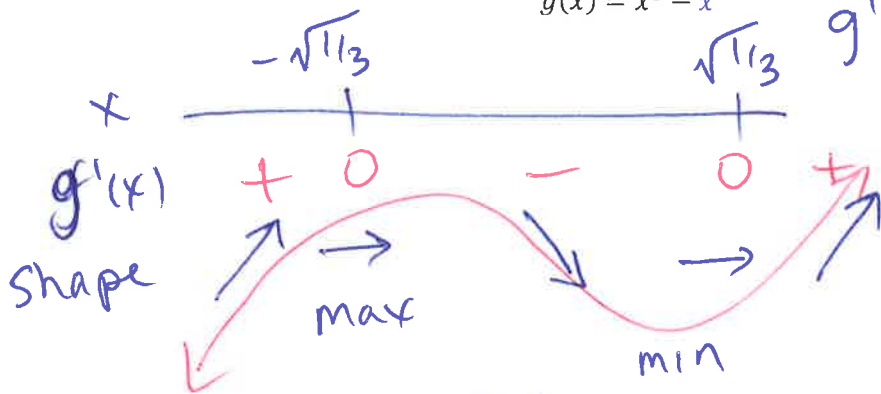
local max @ $x = 0$, $f(0) = -2$

Example: Find the absolute extrema of the curve below on the interval $[-3, 2]$

$$g(x) = x^3 - x$$

$$g'(x) = 3x^2 - 1 = 0$$

$$x = \pm \sqrt{1/3}$$



@ $x = -\sqrt{1/3} \Rightarrow g(-\sqrt{1/3}) = 0.385$ $g(\sqrt{1/3}) = -0.385$

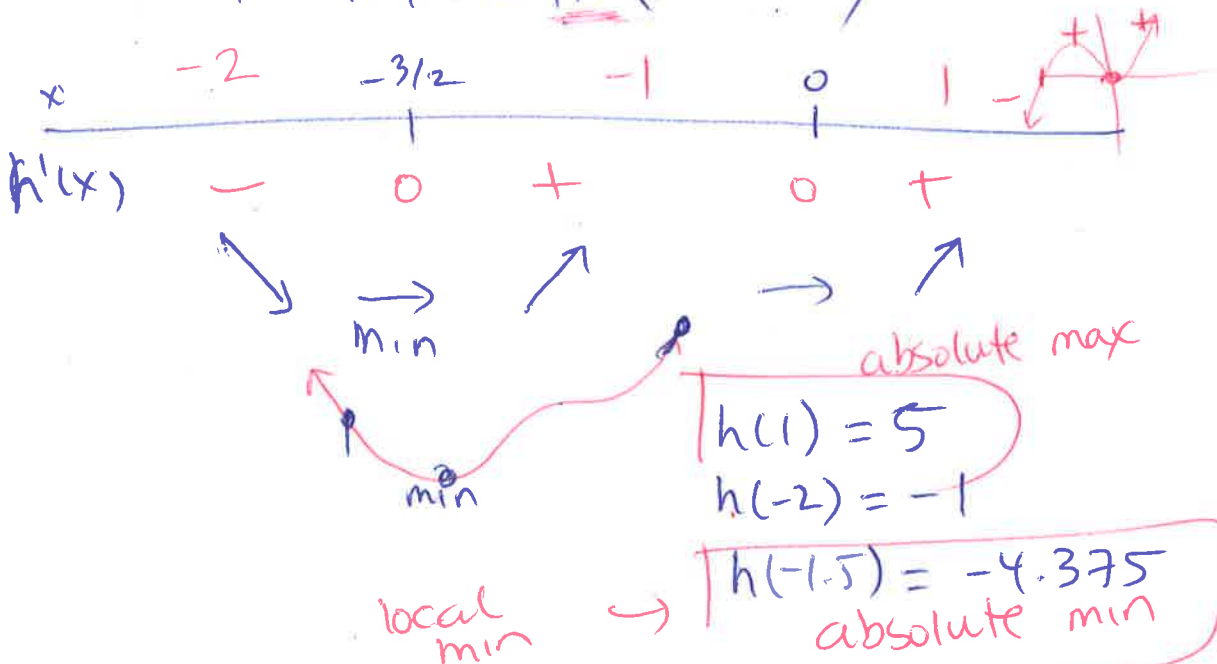
$g(-3) = -24$
 Absolute min

$g(2) = 6$
 Absolute max

Practice: Find all extrema of the curve below on the interval $[-2, 1]$

$$h(x) = 2x^4 + 4x^3 - 1$$

$$h'(x) = 8x^3 + 12x^2 = 4x^2(2x + 3) \quad x = 0, -3/2$$



Practice Problems: 4.2: # 1, 2 and 4 (do what you need), 6-8

4.3: # 1, 5, 6



4.3 # 2

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Terminology:

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- First Derivative Test

Quiz Mond. Test on Feb 4

Review: Find the zeros to the curve, $f(x) = x^4 - 2x - 3$

for large x

$$f'(x) = 4x^3 - 2 = 0 \rightarrow x = \sqrt[3]{1/2}$$

$$x = \frac{-(ANS^4 - 2ANS - 3) + ANS}{4ANS^3 - 2}$$

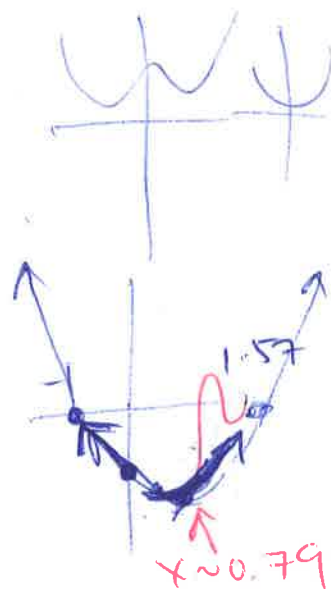
$$z_0 = 1 \rightarrow x = 1.5747 \dots$$

$$z_0 < -1 \rightarrow x = -1$$

$$z_0 = 0 \rightarrow x = -1$$

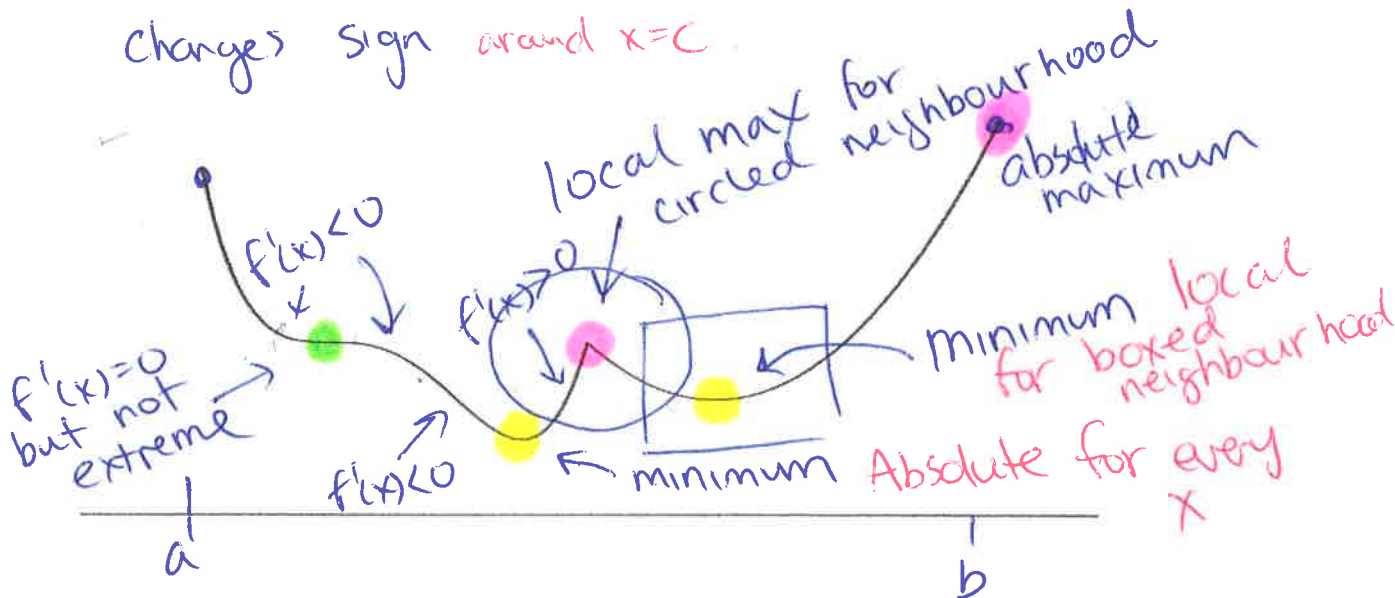
$$z_0 = 0.5 \rightarrow x = -1$$

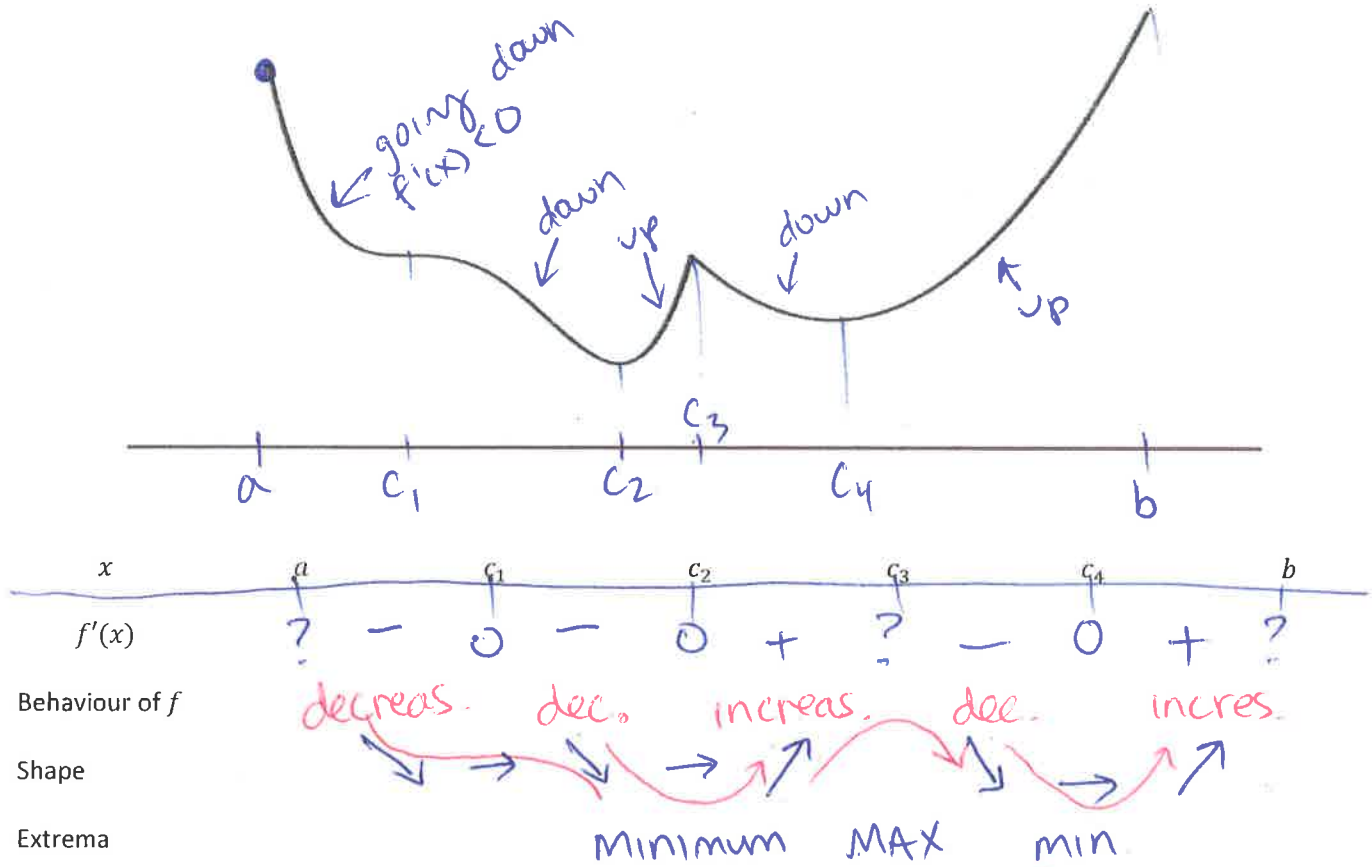
$$z_0 > 2 \rightarrow x = 1.5747$$



What are the conditions for the function to have a maximum or minimum value?

→ a max or min when at a point $x=c$
 $f'(c) = 0$ or is undefined and $f'(x)$
 changes sign around $x=c$.





This is the First Derivative Test:

- Local MAXIMUMs occur when
- Local minimums occur when

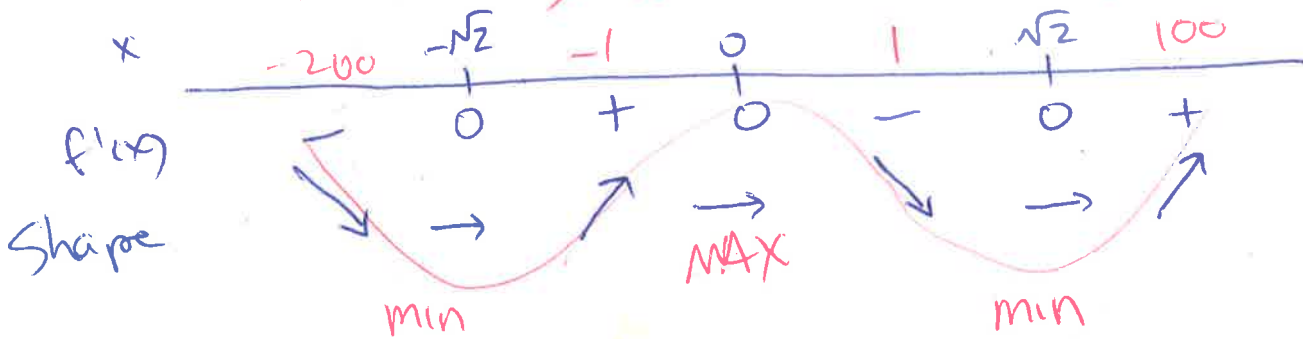
$f'(x)$ goes positive to negative
 $f'(x)$ goes negative to positive

Example: Find the local extrema of the function

$$f(x) = x^4 - 4x^2 - 2, x \in \mathbb{R}$$

$$f'(x) = 4x^3 - 8x = 0$$

$$4x^2(x^2 - 2) = 0 \rightarrow x = 0 \text{ or } \pm\sqrt{2}$$



$$f(-\sqrt{2}) = -6 = f(\sqrt{2})$$

local min
absolute min

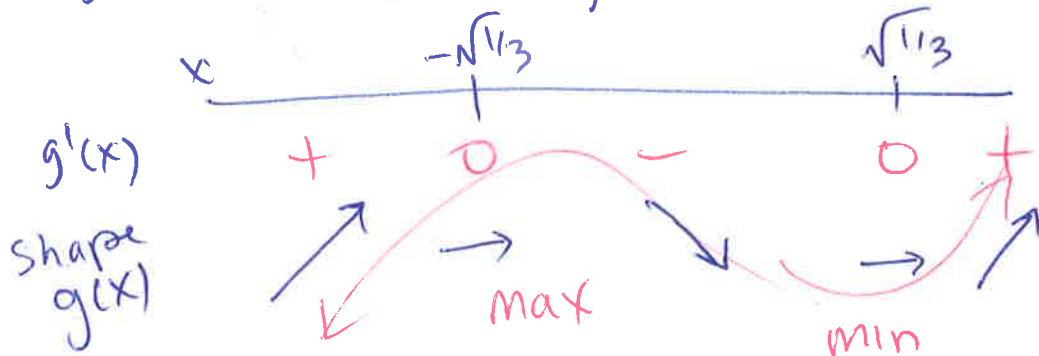
$$f(0) = -2$$

local max

Example: Find the absolute extrema of the curve below on the interval $[-3, 2]$

$$g(x) = x^3 - x$$

$$g'(x) = 3x^2 - 1 \quad g'(x) = 0 \Rightarrow x = \pm\sqrt{1/3}$$



$$g(-\sqrt{1/3}) = 0.385 \text{ local max}$$

$$g(\sqrt{1/3}) = -0.385 \text{ local min}$$

$$g(-3) = -24 \text{ absolute min}$$

$$g(2) = 6 \text{ absolute max}$$

Practice: Find all extrema of the curve below on the interval $[-2, 1]$

$$h(x) = 2x^4 + 4x^3 - 1$$

absolute max $h(1) = 5$

absolute min $h(-1.5) = -4.375$

local max $h(-2) = -1$

Practice Problems: 4.2: # 1, 2 and 4 (do what you need), 6-8

4.3: # 1, 5, 6



4.3 # 2

