Higher Order Differentiation

Goal: • Can use derivative rules (product, quotient, and chain) to find the second, third and n^{th} derivative of a polynomial function. • Can use implicit differentiation to find d^2y/dx^2 • Can use proper notation to describe second, third and n^{th} derivatives
Terminology: • Order of the derivative
Reminder: • Quiz on Thursday November 14 • Test on Wednesday November 20

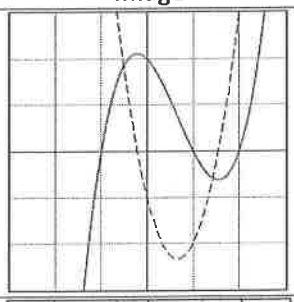
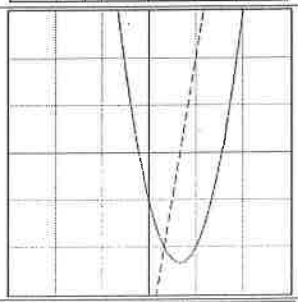
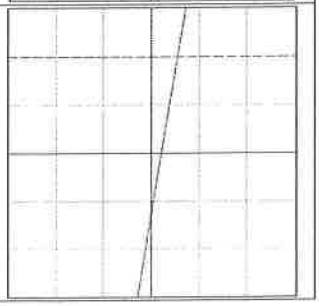
There is nothing new to discuss today. We have talked about derivative rules and how to take a derivative implicitly.

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \frac{d}{dx}(f \cdot g) = g \cdot \frac{df}{dg} + f \cdot \frac{dg}{dx}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g \cdot \frac{df}{dx} - f \cdot \frac{dg}{dx}}{g^2} \qquad \frac{d}{dx}f(u) = \frac{df}{du} \cdot \frac{du}{dx}$$

The big goal of the day is that you can take the derivative of a derivative, but you already did that with earlier review. All I am going to do is formally state the notation for these derivatives.

If $y = f(x)$, then we can describe the derivative of this function:

Derivative Notation	Meaning	Image
$\frac{dy}{dx}$ y'	$\frac{dy}{dx}$ is the change in y as x changes. ⇒ slope (distance)	
$\frac{d}{dx}\left(\frac{dy}{dx}\right)$ y'' $\frac{d^2y}{dx^2}$ $(y')'$	$\frac{d^2y}{dx^2}$ is the change in y' (slope) as x changes ⇒ slope of the slope (velocity)	
$\frac{d^3y}{dx^3}$ y''' $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = (y'')'$	$\frac{d^3y}{dx^3}$ is the change in y'' (slope of the slope) as x changes (acceleration) ⇒ slope of the slope of the slope	

y'''' y^{IIII}

$\sqrt[n]{y}$

$y = \sqrt{x}$

Derivative Notation	Meaning	Image
$\frac{d^n y}{dx^n}$ $y^{(n)}$	the n^{th} derivative (we take the derivative n times)	eventually $y^{(n)} = 0$

* for polynomials

All I want you to do is practice finding derivatives using chain rule and implicit differentiation. Remember that simplifying makes your life easier and we only expand if we plan on simplifying later.

Practice

1. Determine the second derivative rules for:
   a. Power rule

$$\frac{d^2}{dx^2} x^n$$

   b. Product rule

$$(f \cdot g)''$$

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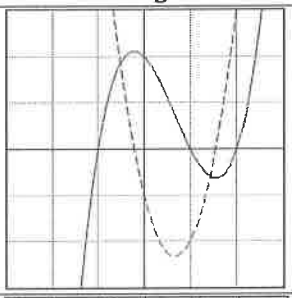
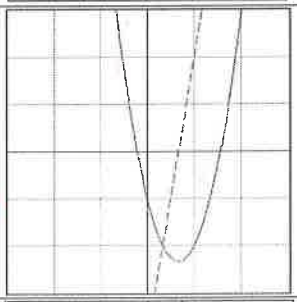
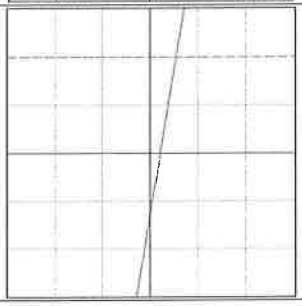
Derivative Notation	Meaning	Image
$\frac{dy}{dx}$ y'	$\frac{dy}{dx}$ is the change in y with respect to x (as x changes) ⇒ slope! (speed)	
$\frac{d^2y}{dx^2}$ y'' $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ $(y')'$	$\frac{d^2y}{dx^2}$ ⇒ is the change in slope (y') as x changes (acceleration) ⇒ slope of the slope	
$\frac{d^3y}{dx^3}$ y''' $((y')')'$	$\frac{d^3y}{dx^2}$ ⇒ is the change in y'' as x changes (jerk) ⇒ slope of the slope of the slope	

y^{IIIII} or y^{IIII} y^5 $y^{IIIIIIIIII}$

y_n y_1 y_{11} y_{111} $\frac{1}{x}$, $\sqrt{x}$

Derivative Notation	Meaning	Image
$\frac{d^n y}{dx^n}$ $y^{(n)}$	$\frac{d^n y}{dx^n}$ or $y^{(n)}$ means take the derivative of y n times (wrt x)	$y^{(n)}=0$ eventually

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