Implicit Differentiation

Goal: • Can describe apply chain rule to find the derivative across functions of different variables and solve for the desired rate of change.
Terminology: • Implicit Differentiation
Reminder: • Quiz on Thursday November 14 • Test on November 20

Review and practice chain rule.

1. Find dy/dx for $y = (3x^2 + 1)^3$

$$y' = \frac{dy}{dx} = \underbrace{3(3x^2+1)^2}_{\text{power rule}} \cdot \underbrace{(6x+0)}_{\text{sum/ power rule}}$$

2. Find dy/dx for $y = (5x^3 - x^4)^7$

$$\frac{dy}{dx} = 7(5x^3 - x^4)^6 (15x^2 - 4x^3)$$

3. Given the following values of x, f, g determine $\frac{d}{dx} f(g(x))$ and $\frac{d}{dx} g(f(x))$ at $x = 0$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	$1/3$
1	3	-4	$-1/3$	$-8/3$

4. Find the equation to tangent line at $x = 3$ for the curve

$$y = \sqrt{x + \sqrt{x^2 + 27}}$$

$$\frac{dy}{dx} = \left(x + (x^2 + 27)^{1/2}\right)^{1/2}$$

← power rule

regular power

power

@ $x=3$ $\quad \left.\frac{dy}{dx}\right|_{x=3} = \frac{1}{4}$

5. For the equation $PV = RT$ where the **changing variables** are P (pressure), V (volume), and T (temperature). In this equation R is a constant and not changing. Find an equation for the change in Pressure as Volume changes.

$$\frac{dP}{dV} = ?$$

$$PV = RT$$

$$\frac{d}{dV}(PV) = \frac{d}{dV}(RT)$$

$$\frac{dP}{dV} \cdot V + \frac{dV}{dV} \cdot P = R \frac{dT}{dV}$$

the rate temp. changes as volume changes

what I want

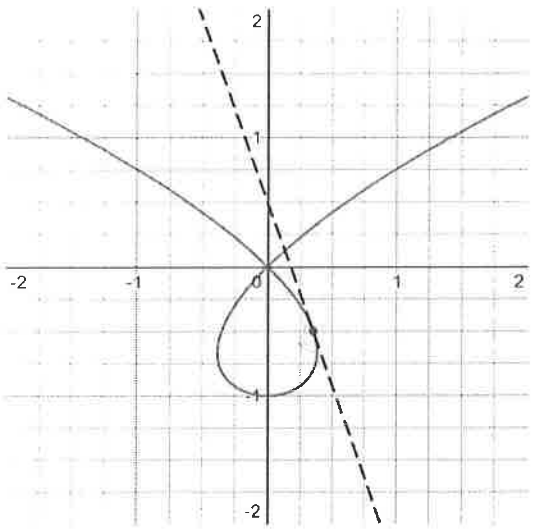
rate vol. changes as vol. changes

$$\frac{dP}{dV} = \frac{R\frac{dT}{dV} - P}{V}$$

We want to introduce one major application of chain rule which number 5 alludes to. Implicit differentiation has to do with finding the slope, dy/dx, of **relations** (not necessarily functions) such as:

What is the slope of the relation $x^2 - y^3 = y^2$ at the point $\left(\frac{1}{\sqrt{8}}, -\frac{1}{2}\right)$

slope = $\frac{dy}{dx}$ ← rate of change as x changes



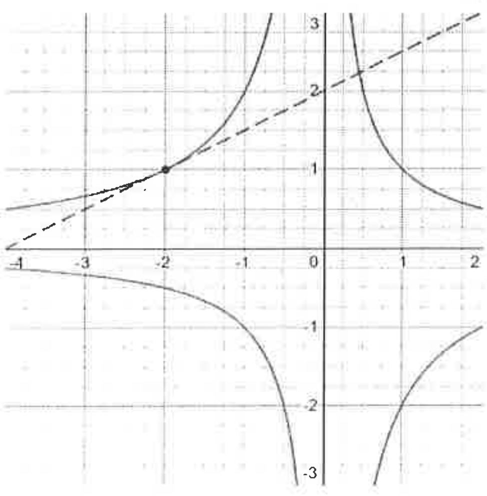
$$\frac{d}{dx}(x^2 - y^3) = \frac{d}{dx} y^2$$

$$2x\frac{dx}{dx} - 3y^2 \cdot \frac{dy}{dx} = 2y \cdot \frac{dy}{dx}$$

$$(2y + 3y^2)\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{2x}{2y + 3y^2} \longrightarrow \text{@ point } \frac{2/\sqrt{8}}{2(-1/2) + 3(1/4)} = -2.828\ldots$$

Practice: What is the slope of the relation $xy + x^2y^2 = 2$ at the point $(-2,1)$



$$\frac{d}{dx}(xy + x^2y^2) = \frac{d}{dx} 2$$

$$\frac{d}{dx}(\overset{f}{x}\overset{g}{y}) + \frac{d}{dx}(x^2y^2) = 0$$

$$\frac{dx}{dx} \cdot y + \frac{dy}{dx} \cdot x + \frac{d}{dx}(x^2) \cdot y^2 + \frac{d}{dx}(y^2) \cdot x^2 = 0$$

$$1 \cdot y + \frac{dy}{dx} \cdot x + 2x \cdot y^2 + 2y\frac{dy}{dx}x^2 = 0$$

$$\frac{dy}{dx} = \frac{-y - 2xy^2}{x + 2x^2y} \longrightarrow \text{@ point slope} = \frac{-1 - 2(-2)(1)^2}{(-2) + 2(4)(1)} = \frac{3}{6} = \frac{1}{2}$$

Practice Problems: 2.7: # 1-3 (do what you need), 4-5 (don't sketch – use Desmos), 8, 9

6, 7, 11

$$\frac{d}{dx}(u^n) = nu^{n-1} \cdot \frac{du}{dx}$$

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Review and practice chain rule.

1. Find dy/dx for $y = (3x^2+1)^3$

$$\frac{d}{dx} g(f(x)) = g'(f(x)) \cdot f'(x)$$

outside $g(x) = x^3$

$f(x) = 3x^2+1$

$g'(f(x)) \cdot f'(x)$

$3(3x^2+1)^2 \cdot (6x)$

2. Find dy/dx for $y = (5x^3 - x^4)^7$

outside $g(x) = x^7$

inside $f(x) = 5x^3 - 4x^4$

$$\frac{dy}{dx} = 7(5x^3 - x^4)^6 \cdot (15x^2 - 4x^3)$$

3. Given the following values of x, f, g determine $\frac{d}{dx} f(g(x))$ and $\frac{d}{dx} g(f(x))$ at $x = 0$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
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4. Find the equation to tangent line at $x = 3$ for the curve

$$y = \sqrt{x + \sqrt{x^2 + 27}}$$

$$\left.\frac{dy}{dx}\right|_{x=3} = \frac{1}{4}$$

outside $g(x) = \sqrt{x}$

inside $f(x) = x + \sqrt{x^2+27}$

new $\frac{d}{dx}\sqrt{x^2+27}$

outside $g_1(x) = \sqrt{x}$

inside $f_1(x) = x^2+27$

5. For the equation $PV = RT$ where the **changing variables** are P (pressure), V (volume), and T (temperature). In this equation R is a constant and not changing. Find an equation for the change in Pressure as Volume changes.

$$\frac{dP}{dV} = ?$$

$$PV = RT$$

$$\frac{d}{dV}(PV) = \frac{d}{dV}(RT)$$

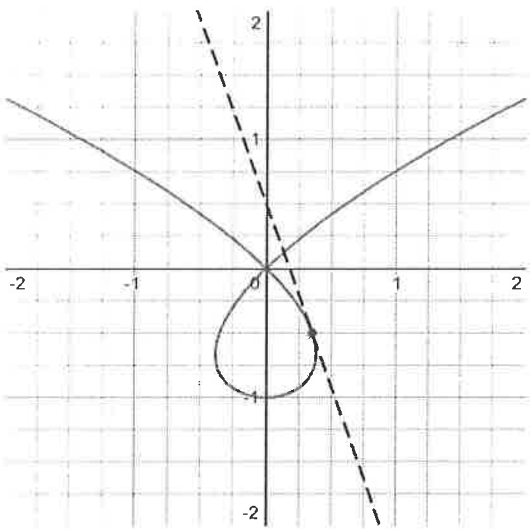
$$\boxed{\frac{dP}{dV}} V + \frac{dV}{dV} \cdot P = R \cdot \frac{dT}{dV}$$

$$\frac{dP}{dV} = \frac{-\frac{dV}{dV}P + R\frac{dT}{dV}}{V}$$

$$= \boxed{\frac{-P + R\frac{dT}{dV}}{V}}$$

We want to introduce one major application of chain rule which number 5 alludes to. Implicit differentiation has to do with finding the slope, dy/dx, of **relations** (not necessarily functions) such as:

What is the slope of the relation $x^2 - y^3 = y^2$ at the point $\left(\frac{1}{\sqrt{8}}, -\frac{1}{2}\right)$ ✱ Slope $= \frac{dy}{dx}$ → based on how much x changes



$$\frac{d}{dx}(x^2 - y^3) = \frac{d}{dx} y^2$$

$$\frac{d}{dx} x^2 - \frac{d}{dx} y^3 = \frac{d}{dx} y^2$$

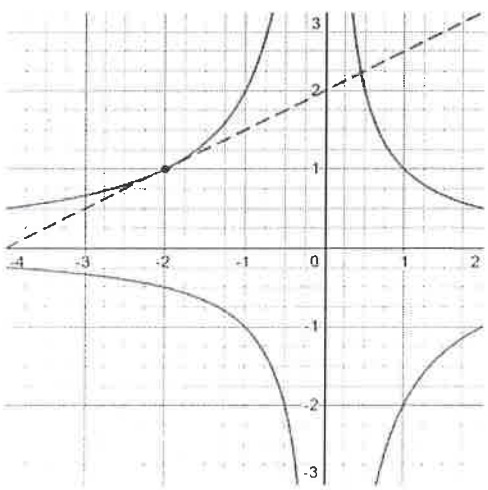
$$2x \cdot \frac{dx}{dx} - 3y^2 \frac{dy}{dx} = 2y \frac{dy}{dx}$$

$$(2y + 3y^2)\frac{dy}{dx} = 2x \frac{dx}{dx}$$

$$\frac{dy}{dx} = \frac{2x \cdot 1}{2y + 3y^2}$$

@ point → $\frac{dy}{dx} = \frac{2(1/\sqrt{8})}{2(-1/2) + 3(1/4)} = -2.83\ldots$

Practice: What is the slope of the relation $xy + x^2y^2 = 2$ at the point $(-2,1)$



$$\frac{d}{dx}(xy) + \frac{d}{dx}(x^2 y^2) = \frac{d}{dx} 2$$

$$\frac{dx}{dx} \cdot y + \frac{dy}{dx} \cdot x + 2x\frac{dx}{dx} \cdot y^2 + 2y\frac{dy}{dx} \cdot x^2 = 0$$

$$\frac{dy}{dx}(x + 2yx^2) = -y - 2xy^2$$

$$\frac{dy}{dx} = \frac{-y - 2xy^2}{x + 2yx^2}$$

@ point → $\frac{dy}{dx} = \frac{1}{2}$

Practice Problems: 2.7: # 1-3 (do what you need), 4-5 (don't sketch – use Desmos), 8, 9

\# 6, 7, 11