Logarithmic Differentiation

Goal:	
	• Can use log laws to take the derivative of very-fast growing functions like x^x
	• Can use log laws to take derivative of massive products and quotients
Terminology:	
	• Logarithmic Differentiation
Reminder:	
	• Quest on Friday

Review: Find y' given the following (you probably want to do this on the board...)

$$y = \ln \sqrt{\frac{(5-x)^3 \cdot \ln(x^2+1)}{e^{6x} \cdot \sqrt[3]{-4x+7}}}$$

$$y = \ln \left[\frac{(5-x)^{3/2} \cdot (\ln(x^2+1))^{1/2}}{e^{3x} \, (-4x+7)^{1/6}} \right]$$

$$\star = \frac{3}{2}\ln(5-x) + \frac{1}{2}\ln(\ln(x^2+1)) - 3x \cancel{\ln e} - \frac{1}{6}\ln(-4x+7)$$

$$y' = \frac{3}{2}\frac{-1}{5-x} + \frac{1}{\cancel{2}}\frac{1}{\ln(x^2+1)} \cdot \frac{1}{x^2+1} \cdot \cancel{2}x - 3 + \frac{4}{6}\frac{1}{(7-4x)}$$

What does this problem teach us? log rules make ~~derivates~~ derivatives of product / quotients very easy.

$\ln(AB) = \ln A + \ln B$

$\ln(A/B) = \ln A - \ln B$

$\ln A^n = n \ln A$

Example: Find dy/dx if

$$\ln\left(y = x^x\right)$$

$$\Rightarrow \ln y = \ln x^x = x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln x + \frac{x}{x} = \ln x + 1$$

extra

$$\frac{dy}{dx} = y(\ln x + 1) = x^x(\ln x + 1)$$

Practice: Find dy/dx if

$$y = x^{2^x}$$

$y(3) = ?$ 3^8 OR 9^3

$x^{(2^x)}$ $(x^2)^x$

$$\ln y = 2^x \ln x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 2^x \ln 2 \cdot \ln x + \frac{2^x}{x}$$

$$\frac{dy}{dx} = 2^x \cdot x^{2^x}\left[\ln 2 \cdot \ln x + \frac{1}{x}\right]$$

$y = 2x + 1$

$y(3) = ?$ 7 OR 8

$(2x)+1$ $2(x+1)$

Practice: Find dy/dx if

$$y = \ln^{\sqrt{x}} x = (\ln x)^{\sqrt{x}}$$

Practice Problems: 8.6: # 1, 2abe, 3, 4

3 but tangent line passes through (2,0).
Solution $y = a^a(\ln a + 1)(x - 2)$, where $a = 2.51971151 \ldots$ or $0.210392243 \ldots$

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$$y = \ln\sqrt{\frac{(5-x)^3 \cdot \ln(x^2+1)}{e^{6x} \cdot \sqrt[3]{-4x+7}}}$$

$$y = \frac{1}{2}\left[\ln(5-x)^3 + \ln[\ln(x^2+1)] - \ln e^{6x} - \ln\sqrt[3]{7-4x}\right)$$

$$= \frac{3}{2}\ln(5-x) + \frac{1}{2}\ln(\ln(x^2+1)) - 3x - \frac{1}{6}\ln(7-4x)$$

$$y' = \frac{-3}{2(5-x)} + \frac{1}{\cancel{2}\ln(x^2+1)} \cdot \frac{1}{(x^2+1)} \cdot \cancel{2}x - 3 + \frac{4}{6(7-4x)}$$

$$\frac{dy}{dx} = \frac{3}{2(x-5)} + \frac{x}{(x^2+1)\ln(x^2+1)} + \frac{2}{3(7-4x)} - 3$$

What does this problem teach us? log laws make the derivative of a product / quotient much simpler.

one-to-one pass horiz./vertical line test

Example: Find dy/dx if

$$\ln(y = x^x)$$

$\Rightarrow \ln y = \ln(x^x)$

$\frac{d}{dx}(\ln y = \underbrace{x}\cdot\underbrace{\ln x})$

$\frac{1}{y}\cdot\frac{dy}{dx} = 1\cdot\ln x + \frac{1}{x}\cdot x = \ln x + 1$

$\Rightarrow \frac{dy}{dx} = y(\ln x + 1) = x^x(\ln x + 1)$

Practice: Find dy/dx if

$$y = x^{2^x}$$

$\ln y = \underbrace{2^x}\cdot\underbrace{\ln x}$

$\frac{1}{y}\frac{dy}{dx} = 2^x\cdot\ln 2\cdot\ln x + 2^x\frac{1}{x}$

$\frac{dy}{dx} = x^{2^x}\cdot 2^x\left[\ln 2\cdot\ln x + \frac{1}{x}\right]$

$y(3) = ?$ $\boxed{3^8}$ OR 9^3

$y = x^{(2^x)}$ OR $(x^2)^x$

$2x+1$

$2(x+1)$

Practice: Find dy/dx if

$$y = \ln^{\sqrt{x}} x$$

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