## One-Sided Limits

## Goal:

- Can determine the value of the limit using left and right-hand approaches
- Can use the definition of continuity alongside piecewise functions


## Terminology:

- Continuous


## Review

Determine the following limits given the graph of $f$


1. $\lim _{x \rightarrow-4} f(x)=$
2. $\lim _{x \rightarrow 1} f(x)=$
3. $\lim _{x \rightarrow 3} f(x)=$

Group: What about $\lim _{x \rightarrow-2} f(x)$ ? Note that $f(x)$ is undefined for $x \in(-3,-2)$.

This gives us another definition of the limit as $x$ approaches $c$.

We are going to use this definition in conjunction with the definition of continuity.
Continuity Definition: A function is continuous at the point $c$ if and only if the following is true.

$$
f(c)=\lim _{x \rightarrow c^{+}} f(x)=\lim _{x \rightarrow c^{-}} f(x)
$$

${ }^{* *}$ Note that this implies two things aside from the obvious that the limit is the value of the function
1.
2.

Example: Determine when the following function is discontinuous (not continuous)

$$
f(x)=\left\{\begin{array}{rr}
1+x, & x<0 \\
\sqrt{1+x}, & 0<x<3 \\
2, & x \geq 3
\end{array}\right.
$$

Practice: Determine when the following function is discontinuous (not continuous) and add statements to make it continuous.

$$
g(x)=\left\{\begin{array}{rr}
(x+2)^{2}, & x \leq-1 \\
2 x+3, & -1<x<4 \\
x+8, & x>4
\end{array}\right.
$$

Practice Problems: 1.3: \# 1-4*, 5-10 (every other), 11, 12, 14

* are warm up questions - do what you need

