Optimization: Distances

Goal :
- Can interpret the zeros of the derivative of some function.
- Can create an equation for geometrically connected obects.

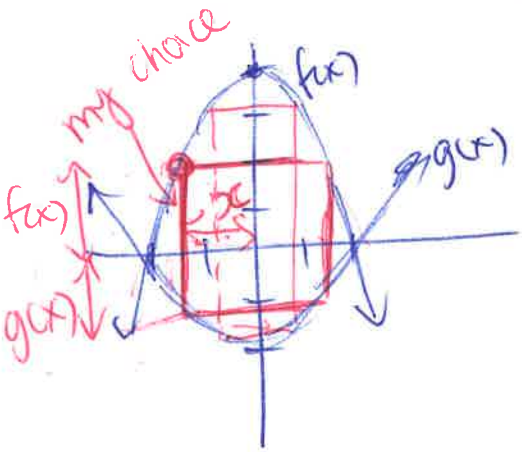
Terminology:
- Optimization

Reminder:
- Test on Feb 4th

We know that local maximum and minimums occur for the function f when f' changes sign (First Derivative Test). Optimization is the application of finding max and minimums in order to maximize material used, or minimize cost to build.

Example: What is the largest rectangle (in terms of area) that can fit between the parabolas?

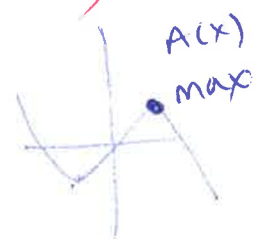
$$y = 4 - x^2 = f(x)$$
$$y = \frac{1}{2}x^2 - 2 = g(x)$$



$$A(x) = \underbrace{(f(x) - g(x))}_{\text{height}}\ \underbrace{2x}_{\text{width}}$$

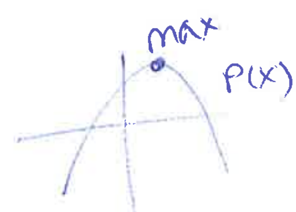
~~width~~

$$= (4 - x^2 - \tfrac{1}{2}x^2 + 2)(2x)$$
$$= (6 - \tfrac{3}{2}x^2)(2x)$$
$$A(x) = 12x - 3x^3$$



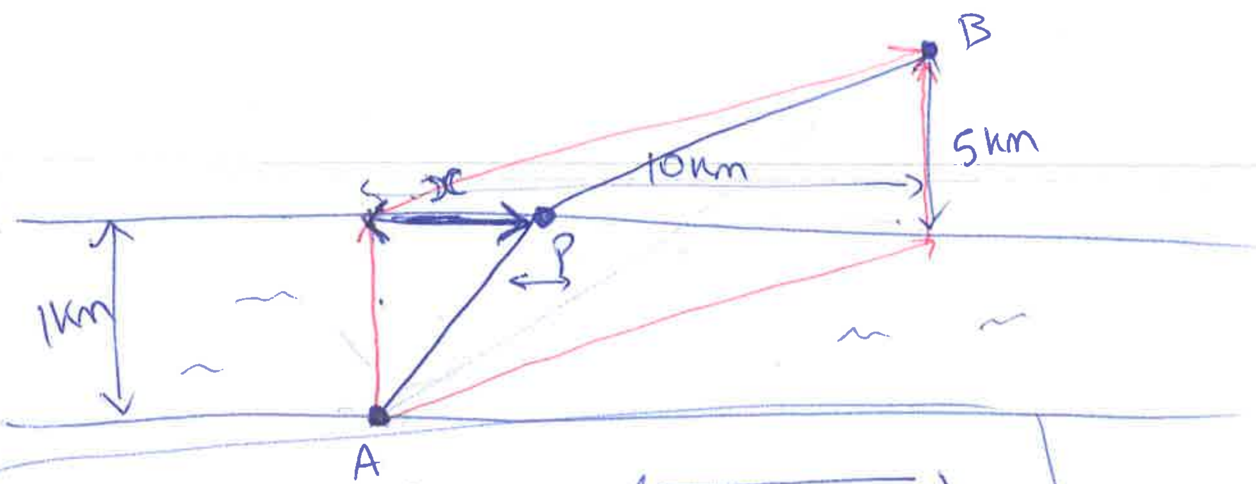
$$A'(x) = 12 - 9x^2 = 0$$
$$\Rightarrow x = \frac{2}{\pm\sqrt{3}} \rightarrow x = \frac{2}{\sqrt{3}}$$ will maximize area

Practice: What is the largest rectangle (in terms of perimeter) that can fit between the parabolas?



$$P(x) = 2(2x) + 2(6 - \tfrac{3}{2}x^2)$$
$$= 4x + 12 - 3x^2$$
$$P'(x) = 4 - 6x = 0 \Rightarrow x = 2/3$$

Example: Fiber optics need to be laid between two communities. Community A is along a river that is 1 km wide, on the opposite side is community B which is 10 km downstream from A and 5 km inland. It costs \$300/m to install fibre optics under the river and \$200/m to install it on land. How far downstream from A should the cables be built?



$$C(x) = \underbrace{\left(\sqrt{1+x^2}\right)300}_{\text{under river}} + \underbrace{\left(\sqrt{25+(10-x)^2}\right)200}_{\text{on land}}$$

$$C'(x) = \frac{3\cancel{00}\,x}{\sqrt{1+x^2}} - \frac{(10-x)\,2\cancel{00}}{\sqrt{25+(10-x)^2}} = 0$$

$$\frac{9x^2}{1+x^2} = \frac{4(10-x)^2}{25+(10-x)^2}$$

1.) cross multiply
2.) expand
3.) collect like terms
4.) Newtons method.

$$\rightarrow 5x^4 - 100x^3 + 721x^2 + 80x - 400 = 0$$

$$x = 0.725$$

Practice Problems: 4.5: # 1, 2, 5, 6, 8-17, 19

18, 20

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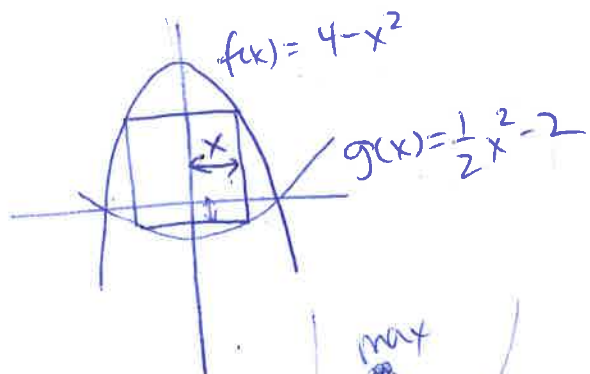
$$y = 4 - x^2$$
$$y = \frac{1}{2}x^2 - 2$$



$A(x) = 2x\left(4 - x^2 - \frac{1}{2}x^2 + 2\right)$

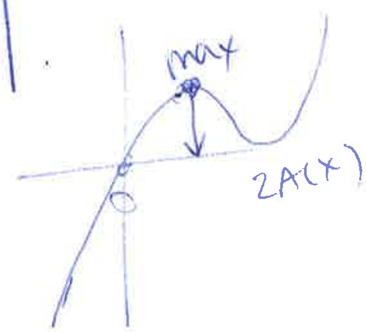
Top - bottom

$= 2x\left(6 - \frac{3}{2}x^2\right)$

$A(x) = 12x - 3x^3$

$A'(x) = 12 - 9x^2 = 0$

$x = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$



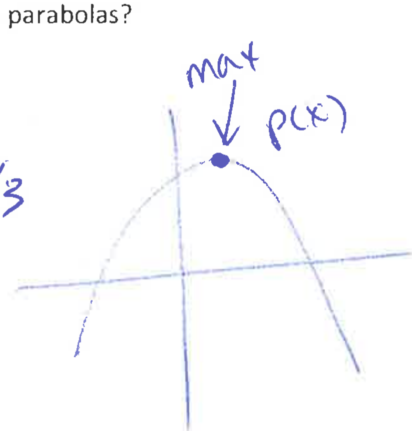
$\frac{4}{\sqrt{3}} \times 4$ is the biggest

Practice: What is the largest rectangle (in terms of perimeter) that can fit between the parabolas?

$P(x) = 4x + 2\left(6 - \frac{3}{2}x^2\right)$

$P'(x) = 4 - 6x = 0 \rightarrow x = 2/3$

$\frac{4}{3}$ by $\frac{16}{3}$



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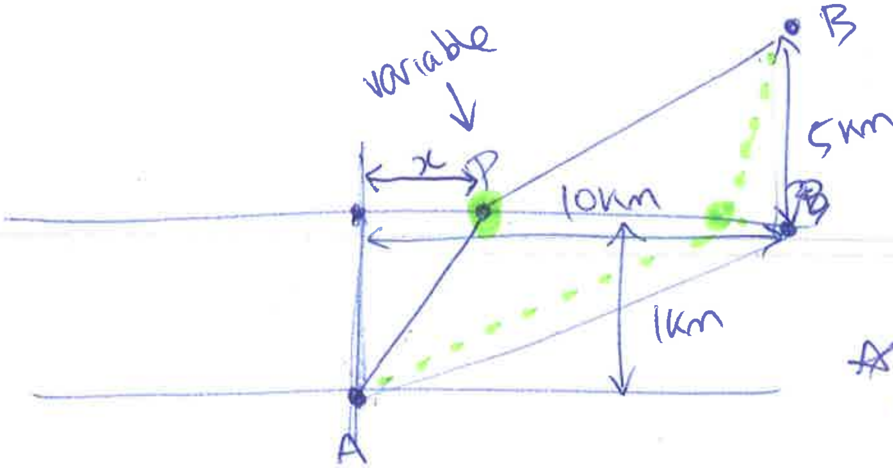

* we need our functions to be in terms of 1 variable.

$C(x) = C_{river}(x) + C_{land}(x)$

cost as distance changes

$= \left(\sqrt{1+x^2}\right)300 + \left(\sqrt{25+(10-x)^2}\right)200$

$C'(x) = \dfrac{300x}{\sqrt{1+x^2}} - \dfrac{(10-x)200}{\sqrt{25+(10-x)^2}} = 0$

$\left(\dfrac{3x}{\sqrt{1+x^2}} = \dfrac{2(10-x)}{\sqrt{25+(10-x)^2}}\right)^2$

$9x^2\left(25+(10-x)^2\right) = 4(10-x)^2(1+x^2)$ ↳ expand

$5x^4 - 100x^3 + 721x^2 + 80x - 400 = 0$ ↳ Newtons method

$x = 0.725$

wed. last question of quiz
practice optimization

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