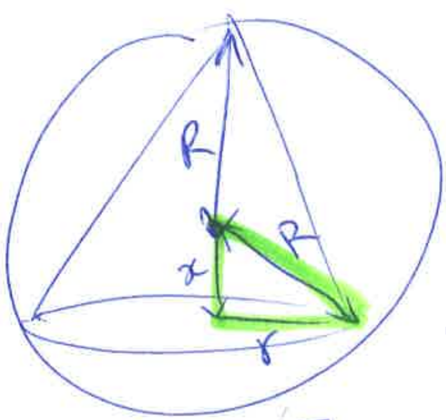
Optimization: Economics

Goal:	• Can interpret the zeros of the derivative of some function. • Can create an equation for geometrically connected objects.
Terminology:	• Revenue • Cost • Demand • Quantity • Marginal • Elasticity
Reminder:	• Test on Feb 4th

Review: What is the dimension of the largest cone that can fit inside a sphere of radius R?



$$x^2 = R^2 - r^2$$

$$h = R + x$$

$$= R + \sqrt{R^2 - r^2}$$

$$V = \frac{\pi}{3}\left[r^2\left(R + \sqrt{R^2 - r^2}\right)\right]$$

$$\frac{dV}{dr} = \frac{\pi}{3}\left[2r\left(R + \sqrt{R^2 - r^2}\right) + r^2 \cdot \frac{-r}{\sqrt{R^2 - r^2}}\right] \overset{set}{=} 0$$

algebra

$$r = \frac{2\sqrt{2}}{3}R$$

Economists like their formulas when considering optimization of profit and cost. Not to get too hung up on the details there are a few important things a business would want to measure:

Characteristic	Dependent on	Example
Revenue earn	quantity / price	$R(q) = -3q^2 + 30q$
Cost	quantity / production cost	$C(q) = q^3 - 6q^2 + 15q$
Profit	Revenue − Cost	$P(q) = R(q) - C(q)$
Demand (Quantity)	price	$q(p) = 10 - \frac{1}{3}p$

q is in 1000s p is in $.

Optimize Profit (ideal quantity output)

$$P(q) = R(q) - C(q)$$

$$P'(q) = 0 \rightarrow \boxed{R'(q) = C'(q)}$$

$$-6q + 30 = 3q^2 - 12q + 15$$

$$3q^2 - 6q - 15 = 0$$

$$q^2 - 2q - 5 = 0$$

$$\boxed{q = 3.45}$$

Optimizing Cost (ideal quantity output)

$$\frac{d}{dq}\boxed{\frac{C(q)}{q}} = 0 \rightarrow \text{average cost} \rightarrow \frac{C'(q) \cdot q - C(q)}{q^2} = 0$$

$$\rightarrow \boxed{C'(q) = \frac{C(q)}{q}}$$

$$3q^2 - 12q + \cancel{15} = q^2 - 6q + \cancel{15}$$

$$2q^2 - 6q = 0$$

$$2q(q-3) = 0$$

$$\boxed{q = 3}$$

Optimizing Revenue (ideal selling price)

$$R(q) = -3q^2 + 30q$$

$\Rightarrow$ Revenue = demand x price

$$R(p) = \left(10 - \tfrac{1}{3}p\right)p$$

$$R'(p) = 10 - \tfrac{2}{3}p = 0$$

$$p = \$15$$

$$\frac{d}{dp}\left(R(p) = q \cdot p\right)$$

$$0 > p\frac{dq}{dp} + q \Rightarrow \frac{dq}{dp} < -\frac{q}{p}$$

$$\underbrace{\boxed{\frac{dq}{dp} \cdot \frac{p}{q}}}_{\text{elasticity} = \varepsilon} < -1$$

if $\varepsilon < -1 \rightarrow \frac{dR}{dp} < 0$

we should $\downarrow p$ to $\uparrow$ revenue

if $\varepsilon > -1 \rightarrow \frac{dR}{dp} > 0$

we should $\uparrow p$ to $\uparrow$ revenue

Practice Problems: 4.5: # 1-3, 5-7

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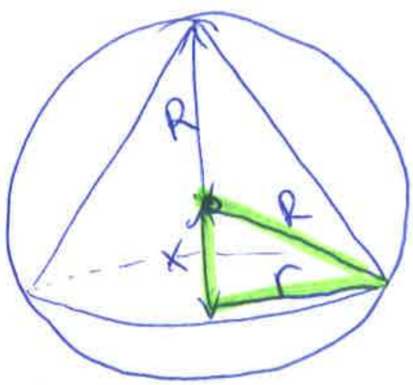

$$x = \sqrt{R^2 - r^2}$$

$$h = R + x = R + \sqrt{R^2 - r^2}$$

$$V = \frac{1}{3}\pi r^2 \left(R + \sqrt{R^2 - r^2}\right) = V(r)$$

$$0 = V'(r) = \frac{1}{3}\pi \left[2r\left(R + \sqrt{R^2 - r^2}\right) + r^2 \frac{-\cancel{2}r}{\cancel{2}\sqrt{R^2 - r^2}} \right]$$

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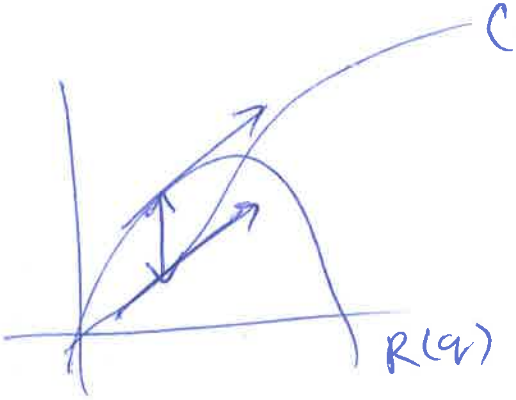
Cost	$\boxed{\text{quantity}}$ / production cost	$C(q) = q^3 - 6q^2 + 15q$
Profit	revenue − cost	$P(q) = R(q) - C(q)$
Demand (Quantity)	price	$q(p) = -\frac{1}{3}(p - 30)$

Optimize Profit (ideal quantity output)

$$P'(q) = 0 \rightarrow \frac{d}{dq}\left(R(q) - C(q)\right) = 0$$

$$\boxed{R'(q) = C'(q)}$$



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Optimizing Cost (ideal quantity output)

$\frac{C(q)}{q}$ ← average cost

optimize

$$\frac{d}{dq}\left(\frac{C(q)}{q}\right) = \frac{C'(q)\cdot q - C(q)}{q^2} = 0$$

$$\boxed{C'(q) = \frac{C(q)}{q}}$$

$$3q^2 - 12q + \cancel{15} = q^2 - 6q + \cancel{15}$$

$$2q^2 - 6q = 0 \rightarrow 2q(q-3) = 0$$

$$\boxed{q = 3}$$

Optimizing Revenue (ideal selling price)

$$\text{Revenue} = \text{price} \times \text{quantity}$$

$$R(p) = p \cdot q(p)$$

$$R'(p) = q(p) + p \cdot q'(p) \gtreqless 0$$

$$q + p \cdot \frac{dq}{dp} \gtreqless 0$$

$$\frac{dq}{dp} \gtreqless -\frac{q}{p} \rightarrow \boxed{\frac{dq}{dp} \cdot \frac{p}{q}} \gtreqless -1$$

elasticity $= \varepsilon$

if $\varepsilon = -1$ we have the best selling price

if $\varepsilon > -1 \Rightarrow R'(p) > 0 \rightarrow$ we should $\uparrow p$ to $\uparrow R(p)$

if $\varepsilon < -1 \Rightarrow R'(p) < 0 \rightarrow$ we should $\downarrow p$ to $\uparrow R(p)$

$$\frac{dq}{dp} \cdot \frac{p}{q} \rightarrow \frac{\Delta q}{\Delta p} \cdot \frac{p}{q} = \frac{\Delta q / q}{\boxed{\Delta p / p}}$$

% change in demand

% change in price

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