Power Rule

Goal:	
	• Can use power rule to take the derivative of monomials
	• Can see the pattern that builds towards the general power rule
Terminology:	
	• Power rule
Reminders	
	• Quiz on Thursday on 2.1-2.3

Review: 1. Given f below, determine $\frac{df}{dx}$ when $x \neq 1$ and $f'(1)$. is undefined

$$f(x) = \begin{cases} x^2, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

$\frac{dF}{dx} = 2x$ ✓ $, x \neq 1$

$f'(1) \neq 2(1)$

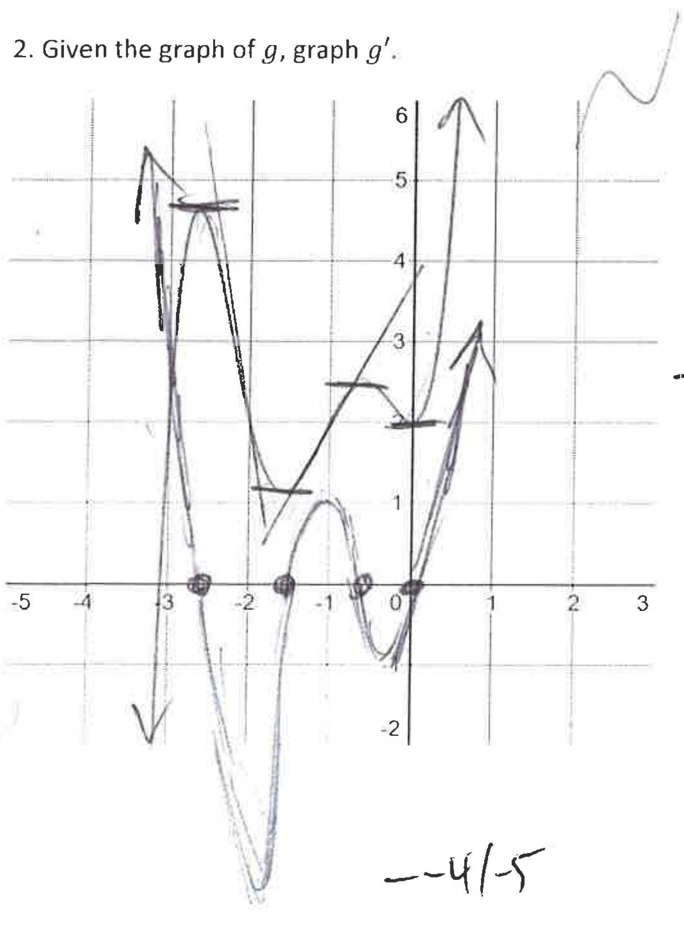
$\neq 2$

$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

$= \lim_{x \to 1} \frac{x^2 - 2}{x - 1}$ $\to -1$ $\to 0^{-/+}$ $\frac{0}{0}$ good

$= DNE$

2. Given the graph of g, graph g'.



→ g is degree 5 b/c there are 4 turns

→ g' is degree 4 b/c there are 3 turns.

We want to be able to take the derivative of powers without having to use a limit every time. On the board work to find $\frac{d}{dx}x^n = A x^{n-1}$

Using limits so we don't have to use limits again.

$$(x+h)^2 = x^2 + 2xh +$$
$$(x+h)^3 = x^3 + 3x^2h + \ldots$$
$$(x+h)^4 = x^4 + 4x^3h + \ldots$$

$$\lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{(\cancel{x^n} + nx^{n-1}\cancel{h} + \ldots + h^n) - \cancel{x^n}}{\cancel{h}}$$

$$= nx^{n-1}$$

$$\frac{d}{dx}x^n = nx^{n-1} \quad \text{Power Rule}$$

1 less degree than before

☆ our explanation only works for $n \in \mathbb{N}$ (natural #) (positive integer)

Function	Derivative by Power Rule	Derivative by Limits
$y = x^1$	$1x^{1-1}$ 1	
$y = x^2$	$2x^{2-1}$ $2x$	
$y = x^3$	$3x^{3-1}$ $3x^2$	
$y = 5x^4$	$4 \cdot 5x^{4-1}$ $20x^3$	☆
$y = \frac{3}{\sqrt{x}}$	$3x^{-1/2}$ $-\frac{1}{2} \cdot 3x^{-1/2-1}$ $\frac{-3}{2}x^{-3/2} =$	☆ $\frac{-3}{2\sqrt{x^3}}$

Practice Problems: 2.2: # 1-2 (do what you need), 4-8, 11 # 9, 12

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Review: 1. Given f below, determine $\frac{df}{dx}$ when $x \neq 1$ and $f'(1)$.

$$f(x) = \begin{cases} x^2, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

$\lim \frac{0}{0}$

$\frac{df}{dx} = 2x$

$f'(1) \neq 2$

$\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

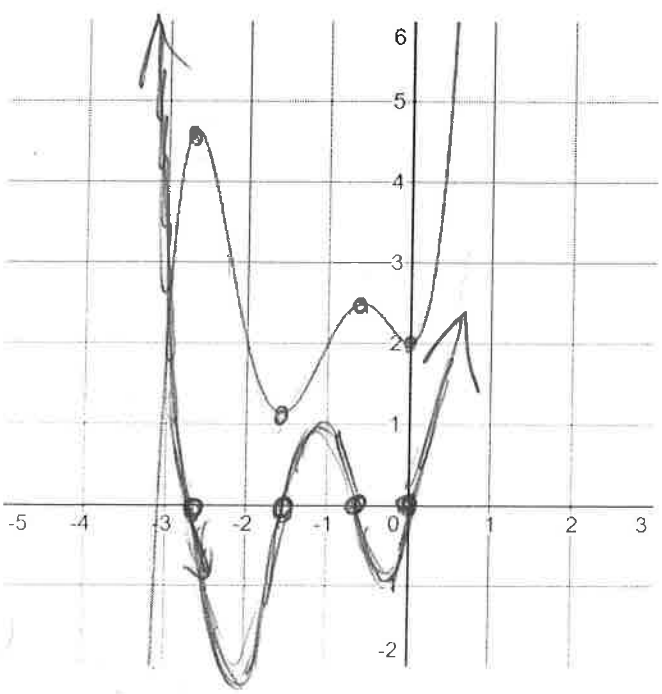
$= \lim_{h \to 0} \frac{(1+h)^2 - 2}{h}$

$\lim_{h \to 0} \frac{1+2h+h^2-2}{h}$

$= \lim_{h \to 0} \frac{h^2+2h-1}{h} \to \frac{-1}{0^{+/-}}$

$= DNE$

2. Given the graph of g, graph g'.



→ the degree of g is 5 b/c there are 4 bumps

→ the degree of g' is 4 b/c there are 3 bumps.

We want to be able to take the derivative of powers without having to use a limit every time. On the board work to find

$$\frac{d}{dx}x^n \approx Ax^{n-1}$$

Using limits so we don't have to use limits again.

$$(x+h)^n = x^n + \dots + h^n$$
$$= x^n + nx^{n-1}h + \dots + h^n$$

b/c
$$(x+h)^2 = x^2 + 2xh + \dots$$
$$(x+h)^3 = x^3 + 3x^2h + \dots$$
$$(x+h)^4 = x^4 + 4x^3h + \dots$$

has an h

$$\lim_{h\to0}\frac{(x+h)^n - x^n}{h} = \lim_{h\to0}\frac{\cancel{x^n} + nx^{n-1}\cancel{h} + \dots + h^{\cancel{n}} - \cancel{x^n}}{\cancel{h}}$$
$$= nx^{n-1}$$

Power Rule

$$\frac{d}{dx}x^n = nx^{n-1}$$

go down in degree
and the coefficient
is the old power.

☆ $n \geq 0$ and an integer

Function	Derivative by Power Rule	Derivative by Limits
$y = x^{1}$	$1x^{1-1}$ $= 1$	
$y = x^2$	$2x^{2-1}$ $2x$	
$y = x^3$	$3x^{3-1}$ $3x^2$	
$y = 5x^4$	$4 \cdot 5x^{4-1}$ $20x^3$	
$y = \frac{3}{\sqrt{x}}$	$3x^{-1/2}$ $-\frac{1}{2} \cdot 3x^{-1/2-1}$ $-\frac{3}{2}x^{-1.5}$	$= -\frac{3}{2\sqrt{x^3}}$

Practice Problems: 2.2: # 1-2 (do what you need), 4-8, 11

9, 12