Product Rule

Goal : • Can use product rule to take the derivative of product of polynomial and radical functions • Understands how to visualize product rule as changing area
Terminology: • Product Rule

We know how to take the derivative of a power and a sum already:

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}(f+g) = \frac{df}{dx}+\frac{dg}{dx}$$

But now we want to take the derivative of a product:

$$\frac{d}{dx}(f \cdot g)$$

Show that the derivative does not distribute through multiplication like it does with addition. That is show

$$\frac{d}{dx}(f \cdot g) \neq \frac{df}{dx}\cdot\frac{dg}{dx}$$

$f(x) = x^2$ $\quad g(x) = 2$

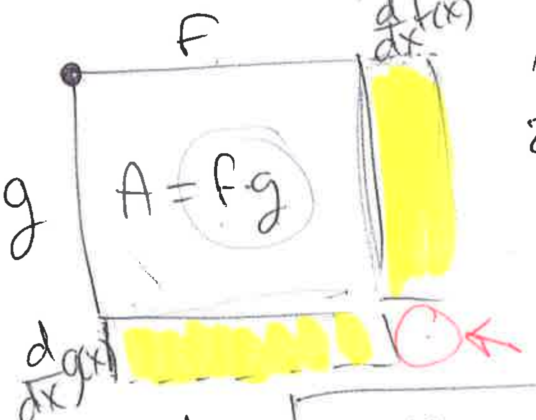
$f'(x) = 2x$ $\quad g'(x) = 0$

$f'(x)\cdot g'(x) = 0$ $\quad$ But $f(x)\cdot g(x) = 2x^2$

$\frac{d}{dx}(2x^2) = 4x$

So, the question remains, what should the derivative be?

$(x+2)(x+3)$



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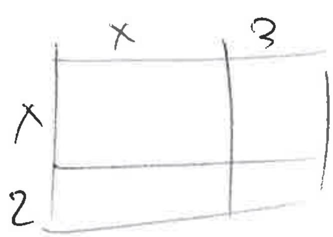

$\frac{d}{dx}A$ → how much does Area change?

$$\frac{d}{dx}A = \boxed{g\cdot\frac{df}{dx} + f\frac{dg}{dx} = \frac{d}{dx}(f\cdot g)} = gf' + fg'$$

Example: Consider the following

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	4	-2	6	3

Determine the equation of the tangent line to the curve $y = f(x) \cdot g(x) + x^2$ at the point $(3, 1)$

$y = f(3) \cdot g(3) + 3^2$
$= 4 \cdot (-2) + 9 = 1$

$y' = (f \cdot g)' + (x^2)'$
$= fg' + gf' + 2x$

$y'(3) = f(3) g'(3) + g(3) f'(3) + 2(3)$
$= 4 \cdot 3 + (-2)(6) + 6 = 6$

$\boxed{y = 6(x-3) + 1}$

Example: Determine the domain of $\frac{dy}{dx}$ and evaluate $y'(1)$ for the function

$$y = \underbrace{(x^3 + 5x^2 - 4x)}_{f}\underbrace{\left(\sqrt{x} - \frac{1}{x}\right)}_{g}$$

$f(x) = x^3 + 5x^2 - 4x$
$f'(x) = 3x^2 + 10x - 4$
$g(x) = \sqrt{x} - \frac{1}{x}$
$g'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{x^2}$

$y' = f'g + g'f$
$= (3x^2 + 10x - 4)\left(\sqrt{x} - \frac{1}{x}\right)$
$+ \left(\frac{1}{2\sqrt{x}} + \frac{1}{x^2}\right)(x^3 + 5x^2 - 4x)$

$y'(1) = (9)(0) + \left(\frac{3}{2}\right)(2) = 3$

Practice Problems: 2.3: # 1-3 (do what you need), 4, 6, 7, 8, 10

9, 11 (do 8 and 10 first)

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$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx}$$

But now we want to take the derivative of a product:

$$\frac{d}{dx}(f \cdot g) = ?$$

Show that the derivative does not distribute through multiplication like it does with addition. That is show

$$\frac{d}{dx}(f \cdot g) \neq \frac{df}{dx} \cdot \frac{dg}{dx}$$

$f(x) = x^2$ $\qquad$ $g(x) = 4x^5$

$f'(x) = 2x$ $\qquad$ $g'(x) = 20x^4$

RHS: $f'(x) \cdot g'(x) = 40x^5$

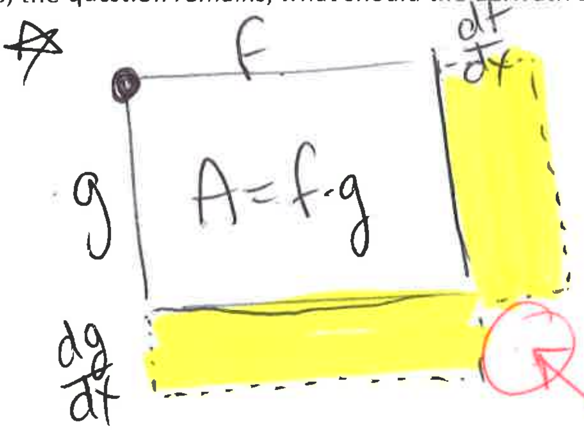
LHS: $\frac{d}{dx}(4x^7) = 28x^6$

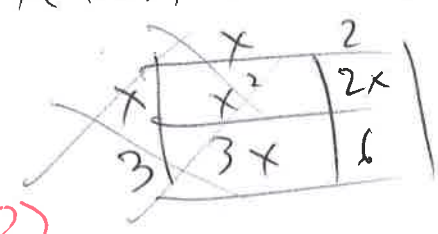
So, the question remains, what should the derivative be?

length width

$(x+2)(x+3)$

$x(x+3) + 2(x+3)$





infintesimly (?) very, very small

$\frac{d}{dx}(fg)$

How the Area Changes as X changes

$$\boxed{\frac{d}{dx}(fg) = \frac{df}{dx} \cdot g + \frac{dg}{dx} f}$$

$(fg)' = f'g + g'f$

Example: Consider the following

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	4	-2	6	3

Determine the equation of the tangent line to the curve $y = f(x) \cdot g(x) + x^2$ at the point $(3, \cancel{+}1)$

$$y' = \frac{d}{dx}\left(f(x) \cdot g(x)\right) + \frac{d}{dx}(x^2)$$

$$= f'(x) \cdot g(x) + g'(x) f(x) + 2x$$

$$= f'(3) g(3) + g'(3) f(3) + 6$$

$$= 6 \cdot (-2) + 3 \cdot 4 + 6 = 6 //$$

$\Rightarrow y = 6(x-3)+1$ tangent line.

Example: Determine the domain of $\frac{dy}{dx}$ and evaluate $y'(1)$ for the function

$$y = \underbrace{(x^3 + 5x^2 - 4x)}_{f(x)}\underbrace{\left(\sqrt{x} - \frac{1}{x}\right)}_{g(x)}$$

$f(x) = x^3 + 5x^2 - 4x$

$f'(x) = 3x^2 + 10x - 4$

$g(x) = \sqrt{x} - \frac{1}{x}$

$g'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{x^2}$

$$y'(x) = f'(x) g(x) + g'(x) f(x)$$

$$= (3x^2 + 10x - 4)\left(\sqrt{x} - \frac{1}{x}\right) + \left(\frac{1}{2\sqrt{x}} + \frac{1}{x^2}\right)(x^3 + 5x^2 - 4x)$$

$$y'(1) = (9)(0) + \left(\frac{3}{2}\right)(2) = 3 //$$

$x \geq 0, x \neq 0$

$\boxed{x > 0}$

Practice Problems: 2.3: # 1-3 (do what you need), 4, 6, 7, 8, 10

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