Quotient Rule

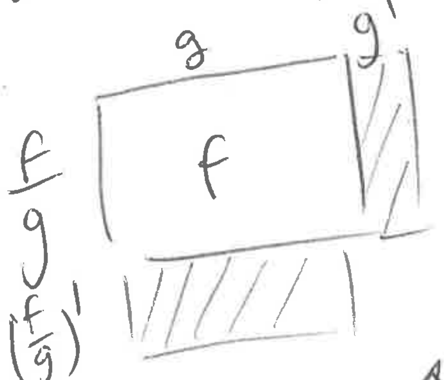
Goal: • Can use quotient rule to take the derivative of quotient of polynomial and radical functions • Understands how to visualize quotient rule as an alternate to product rule
Terminology: • Quotient Rule
Reminder: • Quiz on Monday (all derivative rules covered thus far 2.1-2.5)

Thus far, we have three rules for taking derivatives of polynomial-type functions:

$$\frac{d}{dx}x^n = nx^{n-1}$$
$$\frac{d}{dx}(f \pm g) = \frac{df}{dx} \pm \frac{dg}{dx}$$
$$\frac{d}{dx}(f \cdot g) = f \cdot \frac{dg}{dx} + g \cdot \frac{df}{dx}$$

We illustrated product rule using area of a square. Without using your notes try to recreate the justification for product rule.

see last days notes



$$f' = g'\left(\frac{f}{g}\right) + g\left(\frac{f}{g}\right)'$$
$$\Rightarrow \left(\frac{f}{g}\right)' = \frac{f'}{g} - \frac{g'\left(\frac{f}{g}\right)}{g}$$
$$= \frac{f'g - g'f}{g^2}$$

We want to explore the derivative of a quotient. Using a similar idea as product rule. Illustrate a rule for the derivative of a quotient.

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2}$$

$$\boxed{\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}}$$

★ always take derivative of numerator first.

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

Example: If $f(x) = x^3 - 5x^2 + 8x$ and $g(x) = -2x^4 + 5x - 9$. Then determine $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$

$f(x) = x^3 - 5x^2 + 8x$

$f'(x) = 3x^2 - 10x + 8$

$g(x) = -2x^4 + 5x - 9$

$g'(x) = -8x^3 + 5$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\overset{f'}{(3x^2 - 10x + 8)}\overset{g}{(-2x^4 + 5x - 9)} - \overset{g'}{(-8x^3 + 5)} \cdot \overset{f}{(x^3 - 5x^2 + 8x)}}{\underset{g^2}{(-2x^4 + 5x - 9)^2}}$$

Practice: Determine $h'(1)$ given

$$h(x) = \frac{\overset{f(x)}{(\sqrt{x^3} - 3)(x^2 + 2)}}{\underset{g(x)}{x - \frac{1}{x}}} \quad x \neq 1, 0$$

$f'(x) = \frac{3}{2}x^{1/2}(x^2 + 2) + 2x(\sqrt{x^3} - 3)$

$g'(x) = 1 + \frac{1}{x^2}$

$$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$$

$$h'(x) = \frac{\left[\frac{3}{2}\sqrt{x}(x^2+2) + 2x(\sqrt{x^3} - 3)\right]\left(x - \frac{1}{x}\right) - \left(1 + \frac{1}{x^2}\right)(\sqrt{x^3} - 3)(x^2+2)}{\left(x - \frac{1}{x}\right)^2}$$

$h'(1) =$ undefined b/c can't divide by 0

Practice Problems: 2.5: # 1-3 (do what you need), 4-8

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We want to explore the derivative of a quotient. Using a similar idea as product rule. Illustrate a rule for the derivative of a quotient.

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$$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

Example: If $f(x) = x^3 - 5x^2 + 8x$ and $g(x) = -2x^4 + 5x - 9$. Then determine

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$$

$$f'(x) = 3x^2 - 10x + 8$$

$$g'(x) = -8x^3 + 5$$

$$\frac{d}{dx}\left(\frac{x^3-5x^2+8x}{-2x^4+5x-9}\right) = \frac{\overset{f'}{(3x^2-10x+8)}\overset{g}{(-2x^4+5x-9)} - \overset{g'}{(-8x^3+5)}\overset{f}{(x^3-5x^2+8x)}}{(-2x^4+5x-9)^2}$$

Practice: Determine $h'(1)$ given

$$h(x) = \frac{\boxed{\overset{F}{(\sqrt{x^3}-3)}\overset{G}{(x^2+2)}}\ f(x)}{x - \frac{1}{x}\ \ g(x)}$$

$$f'(x) = F'G + G'F$$

$$g'(x) = 1 + \frac{1}{x^2}$$

$$f'(x) = \frac{3}{2}x^{1/2}(x^2+2) + 2x(\sqrt{x^3}-3)$$

$$h'(x) = \frac{\left[\frac{3}{2}x^{1/2}(x^2+2) + 2x(\sqrt{x^3}-3)\right](x-\frac{1}{x}) - (1-\frac{1}{x^2})\cdot(\sqrt{x^3}-3)(x^2+2)}{(x-\frac{1}{x})^2}$$

$h'(1) =$ undefined b/c divide by 0

not differentiable @ 1

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