Rates of Change Chapter Test: Version A

Name: ______________________________ Date: December 18, 2019

Include all units!

1. Consider the equation for the position of a particle on the x-axis given by

$$x(t) = 10 + 21t - 12t^2 + t^3$$

where t is measured in hours and x is measured in kilometers.

(a) (1 point) What direction is the particle initially moving in? (when $t = 0$).

$v(t) = 21 - 24t + 3t^2$

$v(0) = 21 > 0$ to The right

(b) (1 point) What is the initial acceleration of the particle?

$a(t) = -24 + 6t$

$a(0) = -24\ km/h^2$

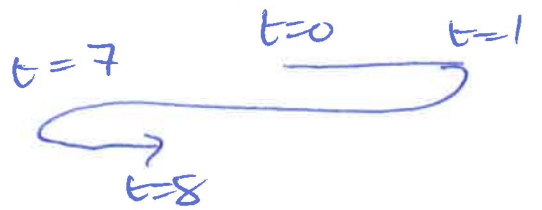
(c) (2 points) What time(s) is the particle stationary?

$v(t) = 21 - 24t + 3t^2 = 0$

$t^2 - 8t + 7 = 0$ $\quad t = 1, \cancel{8}7$ hour

$(t-7)(t-1) = 0$

(d) (3 points) What distance has the particle travelled after 8 hours? Sketch a rough picture of the path the particle has made.



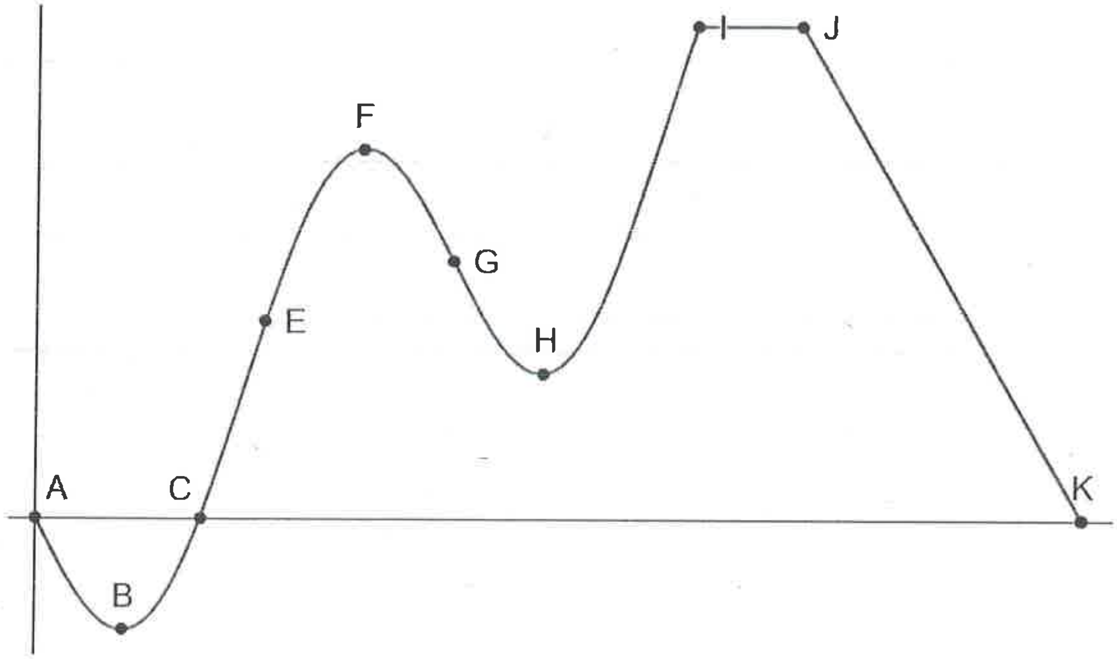
$d = |x(0) - x(1)| + |x(7) - x(1)| + |x(8) - x(7)|$

$= |10 - 20| + |-88 - 20| + |-78 + 88|$

$10 + 108 + 10$

$= \boxed{128\ km}$

2. Consider the following graph of the position of a particle travelling left and right versus time.



(a) (2 points) What characteristic of the graph are you looking at to identify when the particle is moving to the left? When does this occur?

when $v(t) < 0$ (slope is -ve)

A to B , F to H and J to K

(b) (2 points) What characteristic of the graph are you looking at to identify when the particle is accelerating to the right? When does this occur?

when $a(t) > 0$ so velocity (slope) becomes more +ve

A to C , G to I

Rates of Change Chapter Test: Version B

Name: Key Date: December 18, 2019

Include all units!

1. Consider the equation for the position of a particle on the x-axis given by

$$x(t) = 2 - 15t + 9t^2 - t^3$$

where t is measured in hours and x is measured in kilometers.

(a) (1 point) What direction is the particle initially moving in? (when $t = 0$).

$v(t) = -15 + 18t - 3t^2$

$v(0) = -15 \rightarrow$ to the left.

(b) (1 point) What is the initial acceleration of the particle?

$a(t) = 18 - 6t$

$a(0) = 18 \text{ km}^2/h$

(c) (2 points) What time(s) is the particle stationary?

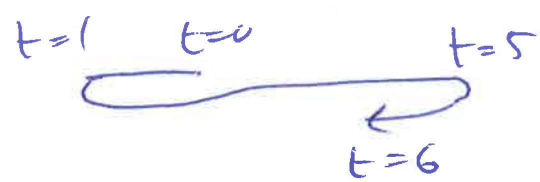
$v(t) = -3t^2 + 18t - 15 = 0$

$t^2 - 6t + 5 = 0$

$(t-5)(t-1) = 0$

$t = 1, 5$ hours

(d) (3 points) What distance has the particle travelled after 6 hours? Sketch a rough picture of the path the particle has made.

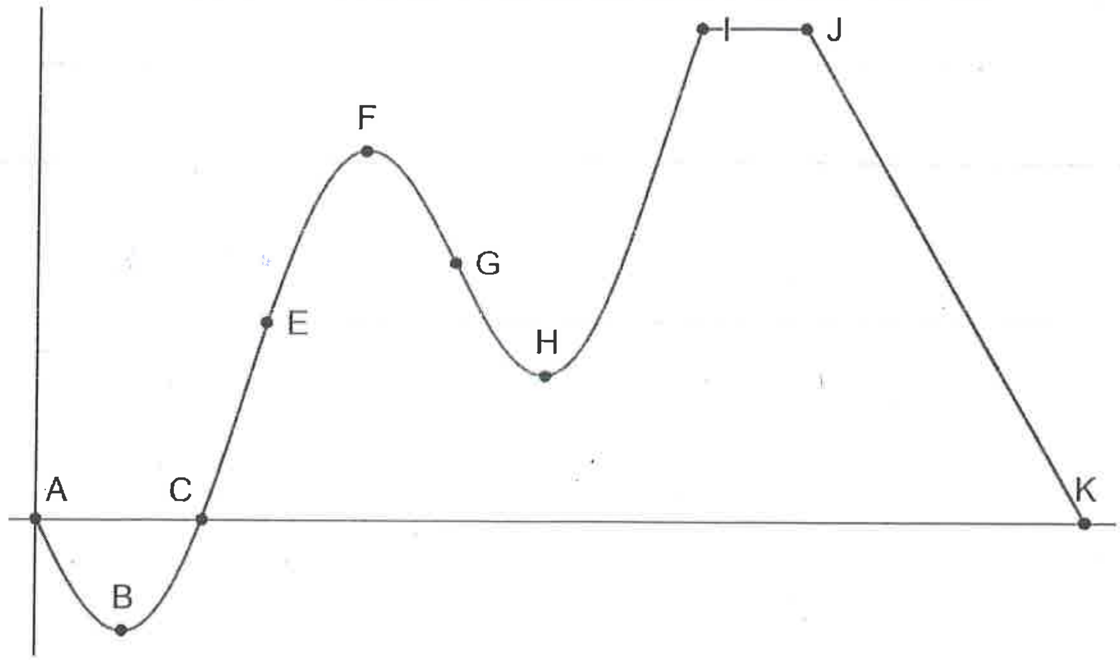


$d = |x(0) - x(1)| + |x(5) - x(1)| + |x(6) - x(5)|$

$= |2+5| + |27+5| + |20-27|$

$= 7 + 32 + 7 = \underline{46 \text{ km}}$

2. Consider the following graph of the position of a particle travelling left and right versus time.



(a) (2 points) What characteristic of the graph are you looking at to identify when the particle is moving to the right? When does this occur?

when slope is +ve

B to F and H to I

(b) (2 points) What characteristic of the graph are you looking at to identify when the particle is accelerating to the left? When does this occur?

when slope is becoming more negative

E to G

Rates of Change Chapter Test: Replacement

Name: ______________________________ Date: January 9, 2019

Include all units!

1. Price, p, and quantity demanded, q, are related by the equation

$$p^3 + \frac{1}{2}q + \frac{1}{2}q^3 = 1$$

Here, both price and demand are represented as numbers between 0 and 1 (percentages).

(a) (2 points) Detemine the change in demand with respect to the change in price.

$\frac{dq}{dp} = ?$

$\frac{d}{dp}\left(p^3 + \frac{1}{2}q + \frac{1}{2}q^3\right) = 0$

$3p^2 + \frac{1}{2}\frac{dq}{dp} + \frac{3}{2}q^2\frac{dq}{dp} = 0$

$$\frac{dq}{dp} = \frac{-3p^2}{\frac{1}{2} + \frac{3}{2}q^2}$$

(b) (3 points) For p and q between 0 and 1, will the rate of change above be positive or negative? What does this tell you about changing the price? That is, if price increases, what happens to the demand? Why is this reasonable?

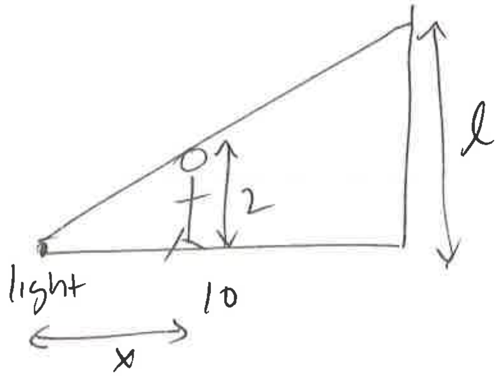
$p, q \in (0,1)$

$\frac{dq}{dp} = \frac{-3p^2 < 0}{\frac{1}{2} + \frac{3}{2}q^2 > 0} \Rightarrow \frac{dq}{dp} < 0 \Rightarrow$ as price increases demand decreases

if something costs more - less ~~ppte~~ people want it

2. A theatre stage has a single light source on the floor 10 m infront of the back curtain of the stage. You are walking directly toward the light and your shadow is on the curtain behind you. In this example you are 2 m tall.

(a) (2 points) Draw and label a picture of the scenario above.



$$\frac{x}{2} = \frac{\cancel{2}\,10}{\ell}$$

(b) (2 points) If you walk toward the light at 0.25 m/s, how fast is the length (or height) of your shadow moving up the curtain when you are halfway between the light and the curtain?

$$x\ell = 20$$

$$\left.\frac{d\ell}{dt}\right|_{x=5}$$

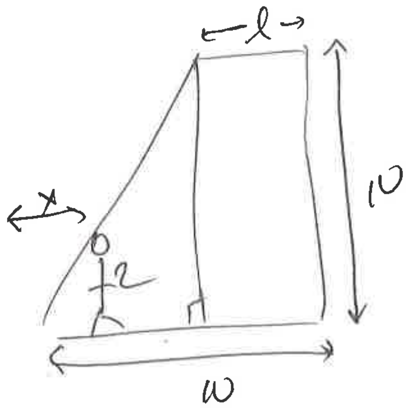
$\cancel{2}$ $\ell = 4$

$$\left.\frac{dx}{dt}\right|_{x=5} = -0.25$$

$$\ell\frac{dx}{dt} + x\frac{d\ell}{dt} = 0$$

$$\frac{d\ell}{dt} = -\frac{\ell}{x}\frac{dx}{dt} \Rightarrow \left.\frac{d\ell}{dt}\right|_{x=5} = \frac{4}{5}\cdot\frac{1}{4} = 0.2\text{ m/s}$$

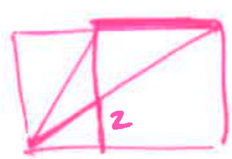
(c) (3 points) If the curtain is only 10 m tall, then once you are 2 m away from the light your shadow will begin to move along the roof (which is parallel to the stage floor). Adjust your drawing so the shadow is moving along the roof, and make a differential equation that relates the length of the shadow on the roof to your position away from the light.



$$\frac{x}{2} = \frac{10-\ell}{10}$$

$$10x = 20 - 2\ell$$

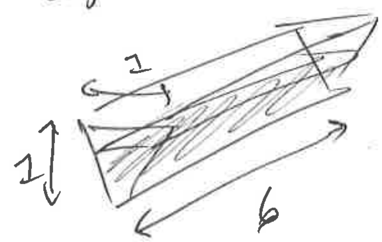
$$-5\frac{dx}{dt} = +\frac{d\ell}{dt}$$



3. Water is being poured into a trough that is in the shape of a triangular prism. The prism is built so the sides are two inverted triangles (point down) whose base and height each measure 1 foot, and the length of the prism is 6 feet. (Volume is area of the triangle (half of base times height) times the length.)

(a) (2 points) Draw and label a picture of the scenario above.

adjust



$V = \frac{1}{2} b h l$

$b = h$

(b) (2 points) If water is being poured into the trough at a constant rate of 2 cubic feet per minute, determine how fast the height of water changes at the moment the height is 0.5 feet.

$\frac{dV}{dt} = 2 \ ft^3/min$ $\qquad \frac{dh}{dt}\Big|_{h=0.5} = ?$

$V = \frac{1}{2} l h^2$ $\qquad \frac{dh}{dt}\Big|_{h=0.5} = \frac{1}{6 \cdot 0.5}(2)$

$\frac{dV}{dt} = l h \frac{dh}{dt}$ $\qquad = \frac{2}{3} \ ft/min$

(c) (1 point) If instead the height is constantly growing at a rate of 0.5 feet per minute, how fast should water be being poured in to the trough? (Leave your answer in terms of the height)

$\frac{dh}{dt} = 0.5 \ ft/min$ $\qquad \frac{dV}{dt} = ?$

$\cancel{\frac{dh}{dt}}$ $\qquad \frac{dV}{dt} = 6h(0.5)$

$\boxed{\frac{dV}{dt} = 3h}$