Rates of Change Rule Quiz

Name: _____ Date: December 4, 2019

Thinking Strategies	Communication	Modelling & Solving

1. A particle is moving along the x-axis with position function

$$x(t) = t^3 - 12t^2 + 45t - 40$$

where t is in minutes and $x(t)$ is the position in μm (micrometers).

(a) Determine the two times and the position of the particle when it is stationary.

$v(t) = 3t^2 - 24t + 45 = 0$

$t^2 - 8t + 15 = 0$

$(t-5)(t-3) = 0$

$t = 3, 5$ min

$x(3) = 14\ \mu m$

$x(5) = 10\ \mu m$

(b) Determine the acceleration at each of these times

$a(t) = 6t - 24$

$a(3) = -6\ \mu m/min^2$ $\quad a(5) = 6\ \mu m/min^2$

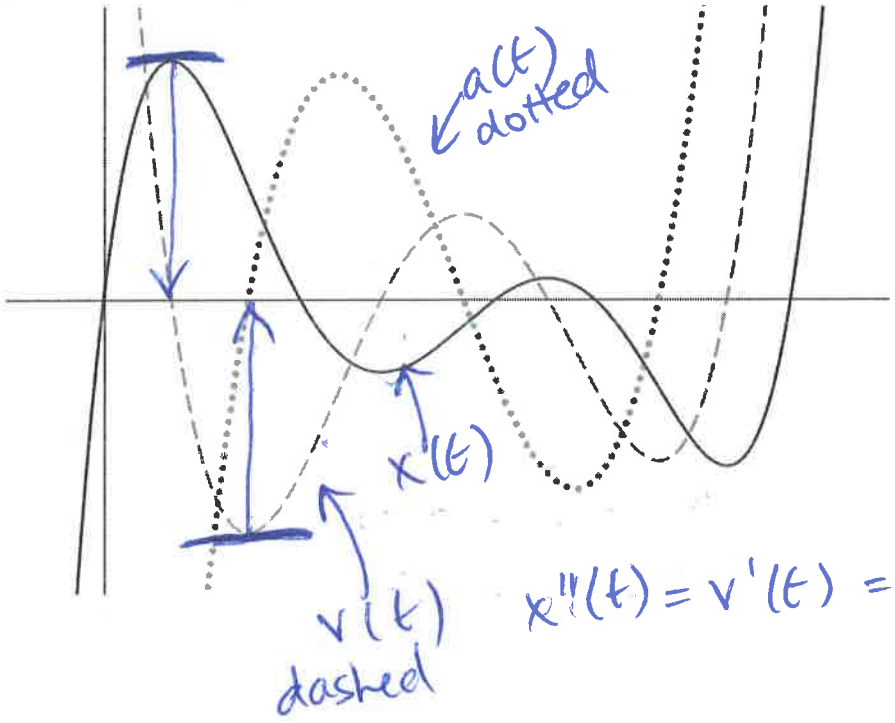
(c) Determine how far the particle has travelled between 2 and 5 minutes.

$d = |x(3) - x(2)| + |x(5) - x(3)|$

$= |14 - 10| + |14 - 10|$

$= 8\ \mu m$

2. Given the three graphs below. Identify which graph describes position, velocity, and acceleration.



$x''(t) = v'(t) = a(t)$

3. The momentum of a system, p, is related to the velocity, v, and mass, m as

$$p = mv$$

Conservation of momentum states that momentum of a system does not change over time.

(a) Relate the change in velocity of a system to the change in mass over time.

$$\frac{d}{dt}(p = mv) \Rightarrow \boxed{0 = \frac{dm}{dt} \cdot v + \frac{dv}{dt} \cdot m}$$

$$\frac{dv}{dt} = \frac{-v}{m}\frac{dm}{dt}$$

(b) Use your differential equation to describe what happens in an explosion (mass changes).

$\frac{dm}{dt} < 0$ mass changing $\Rightarrow$ $\frac{dv}{dt} = \frac{-v}{m}\frac{dm}{dt} > 0$ velocity should ↑

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1. A particle is moving along the x-axis with position function

$$x(t) = t^3 - 12t^2 + 36t - 10$$

where t is in minutes and $x(t)$ is the position in μm (micrometers).

(a) Determine the two times and the position of the particle when it is stationary.

$v(t) = 3t^2 - 24t + 36 = 0$

$t^2 - 8t + 12 = 0$

$(t-6)(t-2)$

$t = 2, 6$ min

$x(2) = 22\,\mu m$

$x(6) = -10\,\mu m$

(b) Determine the acceleration at each of these times

$a(t) = 6t - 24$

$a(2) = -12\,\mu m/min^2$ $\quad a(6) = 12\,\mu m/min^2$

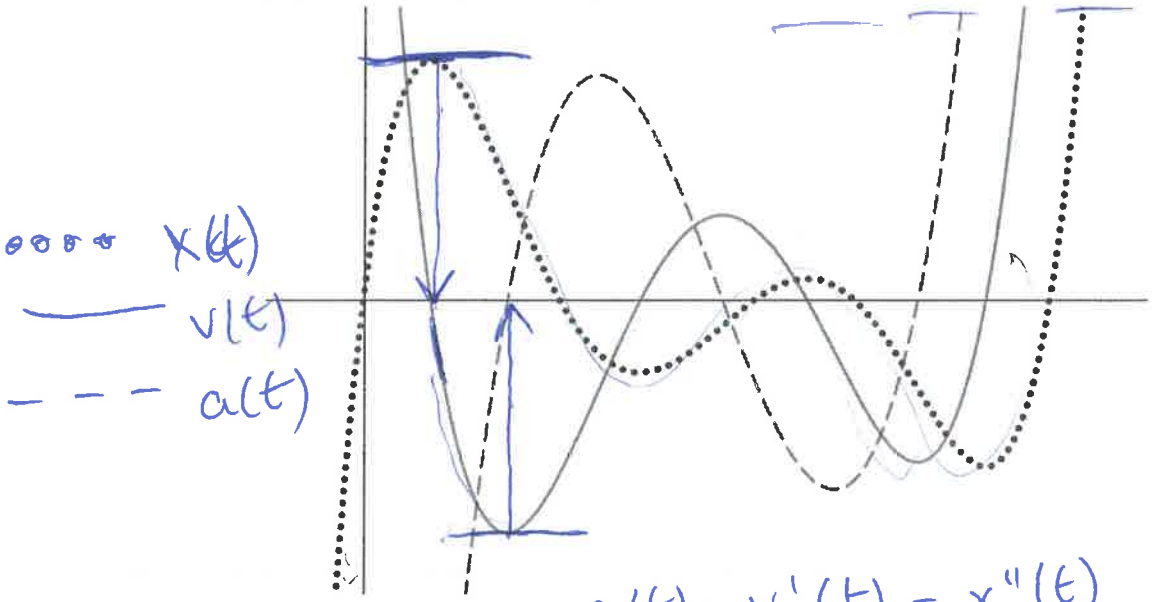
(c) Determine how far the particle has travelled between 1 and 6 minutes.

$d = |x(2) - x(1)| + |x(6) - x(2)|$

$= |22 - 15| + |-10 - 22|$

$= 7 + 32 = 39\,\mu m$

2. Given the three graphs below. Identify which graph describes position, velocity, and acceleration.



3. The momentum of a system, p, is related to the velocity, v, and mass, m as

$$p = mv$$

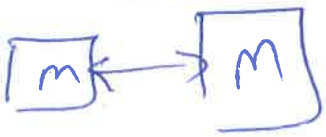
Conservation of momentum states that momentum of a system does not change over time.

(a) Relate the change in velocity of a system to the change in mass over time.

$$\frac{dp}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt} = 0 \Rightarrow \boxed{\frac{dv}{dt} = -\frac{v}{m}\frac{dm}{dt}}$$

(b) Use your differential equation to describe what happens in a collision (mass changes).

$\frac{dm}{dt} > 0$ change in mass



$\frac{dv}{dt} < 0$

$$\frac{dv}{dt} = \frac{-v}{m}\frac{dm}{dt} < 0$$

↑ decreases