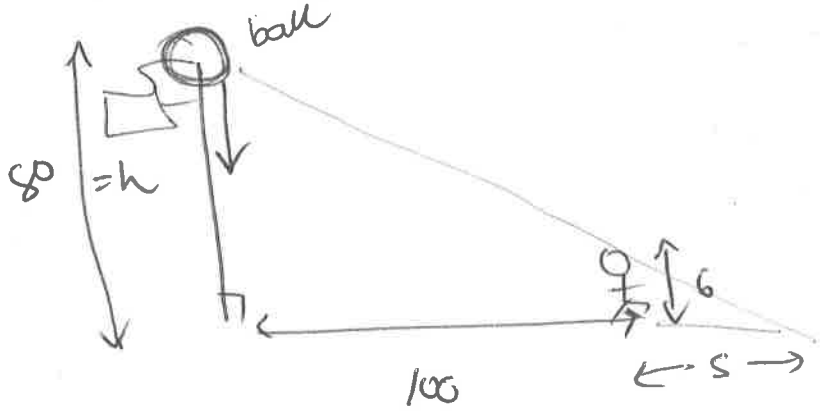
Related Rates Quiz

Name: _____ Date: December 12, 2019

Thinking Strategies	Communication	Modelling & Solving

1. In New York on New Year's Eve, a giant ball of light will drop down a flag pole as people watch. The pole is 80 feet tall and you are 100 feet from the base of the flagpole. Assume your height is 6 feet.

   (a) Draw and label a picture to model this situation.



   (b) If the ball is dropping at rate of 1.3 feet/sec, how fast is the length of your shadow growing when the light is midway down the pole? Assume nothing is behind you to hinder your shadow.

$$\frac{s}{6} = \frac{100+s}{h}$$

$$sh = 600 + 6s$$

$$40s = 600 + 6s$$

$$34s = 600$$

$$s = 17.65$$

$$\Rightarrow \frac{ds}{dt}\cdot h + \frac{dh}{dt}\cdot s = 6\frac{ds}{dt}$$

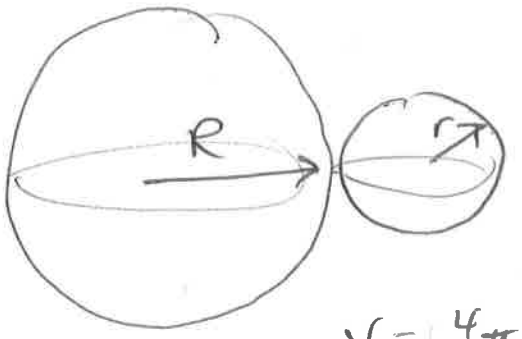
$$\frac{ds}{dt} = \frac{s\frac{dh}{dt}}{6-h}$$

$$\left.\frac{ds}{dt}\right|_{h=40} = \frac{17.65(-1.3)}{6-40}$$

$$= 0.675 \text{ ft/sec}$$

2. Two spherical balloons are connected so that air can move between them, but the total volume is constant. If one balloon changes shape, then the other will change accordingly.

(a) Draw and label a picture to model this situation.



$$V = \frac{4}{3}\pi\left[R^3 + r^3\right]$$

(b) Determine how fast the radius of the smaller balloon is changing if the larger balloon's radius is shrinking 3 cm/sec at the moment the larger radius is two times the size of the smaller radius. Note that volume of a sphere is $V = \frac{4}{3}\pi r^3$.

$$\frac{dR}{dt} = -3 \quad @ \quad R = 2r$$

$$\frac{dV}{dt} = 0 = \frac{4}{3}\pi\left[3R^2\frac{dR}{dt} + 3r^2\frac{dr}{dt}\right)$$

$$\Rightarrow \quad \frac{dr}{dt} = -\frac{R^2}{r^2}\frac{dR}{dt}$$

$$\left.\frac{dr}{dt}\right|_{R=2r} = \frac{-4r^2}{r^2}(-3)$$

$$= 12 \text{ cm/sec}$$