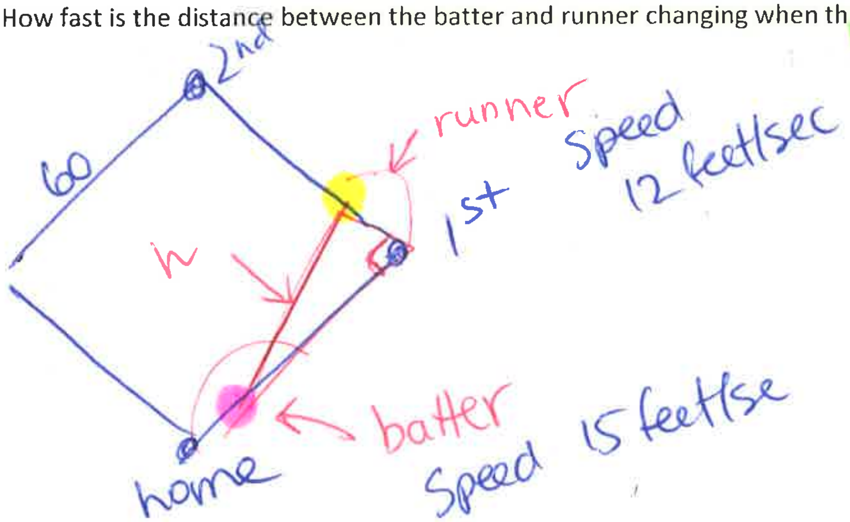
Related Rates

Goal: • Can create an equation to model problems based in geometry – Pythagoras and similar triangles are everything • Can differentiate any equation and relate how the rates depend on each other and make a new differential equation.
Terminology: • Related Rate
Reminder: • Test at the end of the month Dec 18th

We are going to take all of the scenarios you made before and now work backwards. Here is a problem I want you to model, dertermine an equation that relates objects in it and take an appropriate derivative.

Example: A baseball diamond has 4 bases that make a diamond with edge lengths of 60 feet. There is a runner on first when the batter hits a fair ball. The batter runs to first at 15 feet/sec and the runner on first runs to second at 12 feet/sec. How fast is the distance between the batter and runner changing when the batter is 30 feet from first?

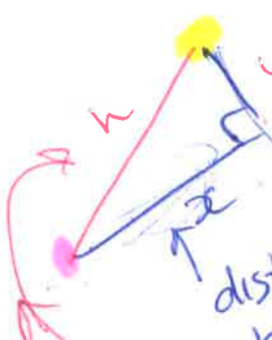


Strategy

1.) Draw a picture
add labels.

2.) Find a relationship
geometric

3.) Take deriv.
and plug in values



$y \rightarrow$ dist from runner + first $\quad \frac{dy}{dt} = 12$

$\Rightarrow \frac{dx}{dt} = -15$

dist from batter + first

$\frac{d}{dt}\ \boxed{x^2 + y^2 = h^2}$ $\qquad \frac{dh}{dt} = ??$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2h\frac{dh}{dt}$

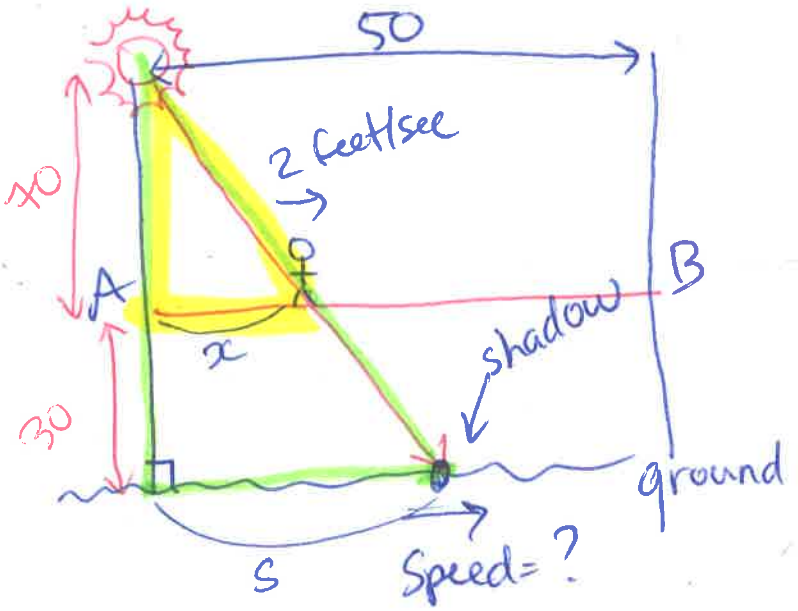
$\Rightarrow \boxed{\frac{dh}{dt} = \frac{x}{h}\frac{dx}{dt} + \frac{y}{h}\frac{dy}{dt}}$

$x = 30 \quad y = 24 \quad h = 38.4$

$\frac{dx}{dt} = -15 \quad \frac{dy}{dt} = 12$

$= -4.2$ ft/sec

Example: A tightrope is stretched 30 feet above the ground between the two buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A. How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the building?

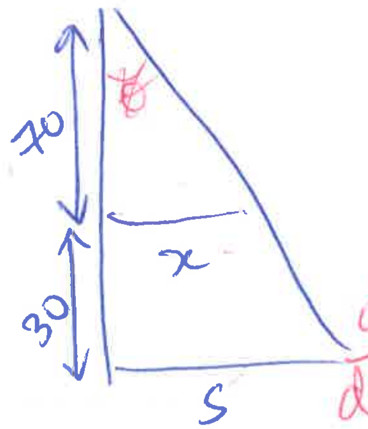


Strategy

1.) draw a picture + label.

2.) relationship.
Similar Δs

3.) Take deriv + plug in values



$\frac{dx}{dt} = 2 \qquad \frac{ds}{dt} = ???$

Shadow velocity

$$\frac{d}{dt}\left(\boxed{\frac{x}{70} = \frac{s}{100}}\right) = \frac{\text{Base}}{\text{height}}$$

$$\frac{1}{70}\frac{dx}{dt} = \frac{1}{100}\frac{ds}{dt} \Rightarrow \frac{ds}{dt} = \frac{10}{7}\frac{dx}{dt}$$

$$= \frac{20}{7}$$

$$= 2.86 \text{ ft/sec}$$

Speed of S doesn't depend on position x

Practice Problems: 3.5: 2D # 1-3, 5, 8-15
3D # 4, 6, 7, 16, 17

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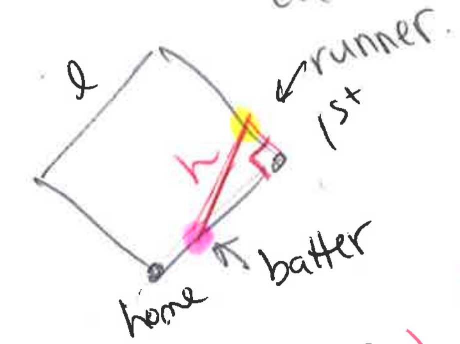
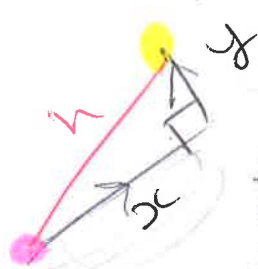
l ~~a~~ between plates 60 feet

$\frac{dl}{dt} = 0$

$V_b = 15$ feet/sec　$V_r = 12$ feet/sec

let h be dist. between batter/runner　$\frac{dh}{dt} = ?$ @ the time batter is 30ft from home.





$\frac{dx}{dt} = -15$

$\frac{dy}{dt} = 12$

$\frac{d}{dt}\left(h^2 = x^2 + y^2\right)$

$\cancel{2}h\frac{dh}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}$

$$\boxed{\frac{dh}{dt} = \frac{x}{h}\frac{dx}{dt} + \frac{y}{h}\frac{dy}{dt}}$$

Strategy

1.) list variables

1.*) draw and label a picture

2.) look for right-angle ~~tria~~ triangles (geometry)

3.) Took derivative + plug in values

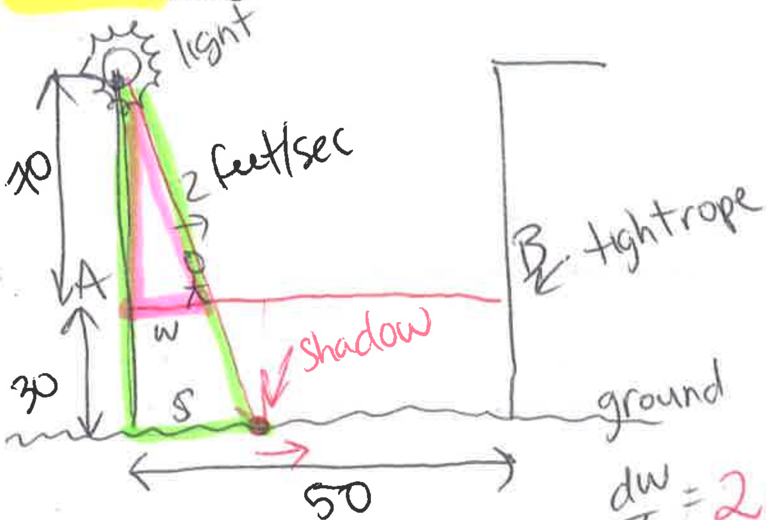
$x = 30$　$y = 24$

$h = 38.4$

$\Rightarrow \frac{dh}{dt} = \frac{30}{38.4}(-15) + \frac{24}{38.4}(12)$

$= -4.2$ feet/sec

Example: A tightrope is stretched 30 feet above the ground between the two buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A. How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the building?



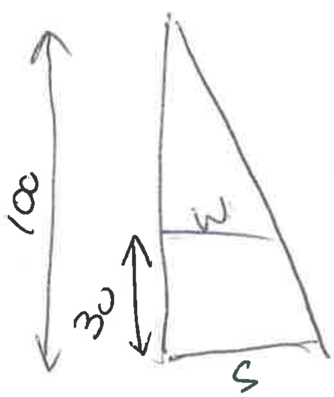
Strategy

1.) Draw/label picture

2.) Found a △ ⇒ similar

3.) Take deriv. and plug in values

$\frac{dw}{dt} = 2$ $\quad \frac{ds}{dt} =$ → how fast the shadow moves



$$\frac{d}{dt}\left(\frac{s}{100} = \frac{w}{70}\right)$$

$$\boxed{\frac{1}{100}\frac{ds}{dt} = \frac{1}{70}\frac{dw}{dt}}$$

← only depends on $\frac{dw}{dt}$

$$\frac{ds}{dt} = \frac{10\cancel{0}}{7\cancel{0}}\frac{dw}{dt} = \frac{10}{7}(2)$$

$$= 2.86 \text{ feet/sec.}$$

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3D # 4, 6, 7, 16, 17