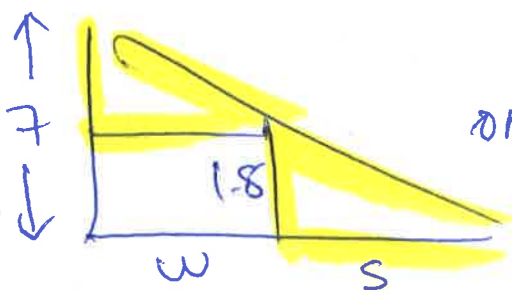
Related Rates

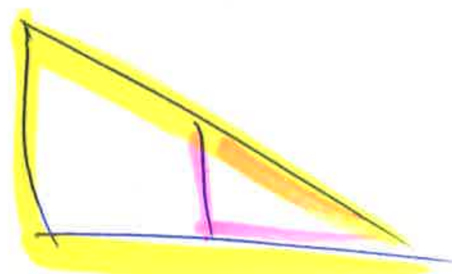
Goal : • Can model problems involving volume and surface area.	
Terminology: • None	
Reminder: • Quiz next week on Dec 13th on Related Rates • Test at the end of the month Dec 18th	

We are going to continue practicing related rates for the next two days

Review: A person, who is 1.8 m tall, is walking 1 m/s down a street at night toward a lightpost that stands 7 m tall. How fast is the length of the person's shadow changing when the person is 5 m away from the lightpost?



OR



$$\frac{dw}{dt} = -1$$

$$\boxed{\frac{ds}{dt} = \frac{1.8}{5.2}\frac{dw}{dt}}$$

$$= -0.346 \, m/s$$

decreasing in size

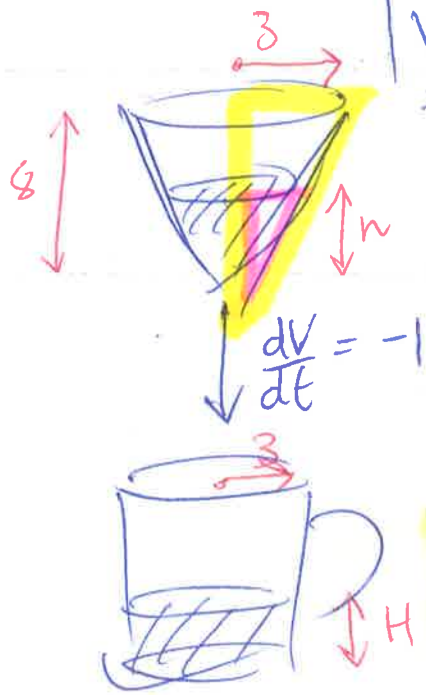
Example: Water is flowing out of a conical fiter into a cylindrical cup at a constant rate of 1 cm^3/sec. If the filer has a height of 8 cm and diameter of 6 cm and the cylindrical cup has the same diamter determine how fast the height of water in the filter and cup are changing.

$$\boxed{V=\frac{1}{3}\pi r^2 h}$$

1.) Draw a picture + label it

2.) find a relationship

3.) Take deriv. + plug in values.



$\frac{dh}{dt}=?$

$\frac{dV}{dt}=-1\,cm^3/s$

$\frac{dH}{dt}=?$ $V=\pi r^2 \cancel{H} = 9\pi H$

$\frac{3}{8}=\frac{r}{h} \Rightarrow \frac{3}{8}h=r$

① $V=\frac{1}{3}\pi\left(\frac{3}{8}h\right)^2 h$

② $\frac{d}{dt}\left(\frac{3}{8}h=r\right)$

$\frac{d}{dt}\left(V=\frac{1}{3}\pi r^2 h\right)$

sub..

$\frac{d}{dt}\left(V=\frac{3\pi}{64}h^3\right)$

$-1=\frac{dV}{dt}=\frac{9\pi}{64}h^2\frac{dh}{dt} \Rightarrow \boxed{\frac{dh}{dt}=\frac{-64}{9\pi h^2}}$ as $h\to 0$ $\left|\frac{dh}{dt}\right|\to\infty$

$\frac{d}{dt}\left(V=9\pi H\right) \Rightarrow \frac{dV}{dt}=9\pi\frac{dH}{dt} \Rightarrow \boxed{\frac{dH}{dt}=\frac{1}{9\pi}}$ → const.

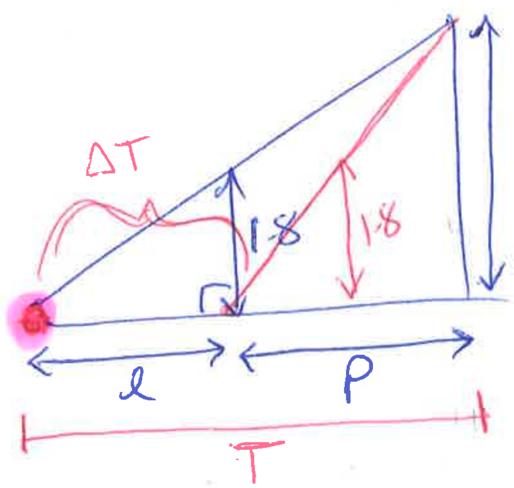
Practice Problems: 3.5: 2D # 1-3, 5, 8-15
3D # 4, 6, 7, 16, 17

Related Rates

Goal : • Can model problems involving volume and surface area.
Terminology: • None
Reminder: • Quiz next week on Dec 13th on Related Rates • Test at the end of the month Dec 18th

We are going to continue practicing related rates for the next two days

Review: A person, who is 1.8 m tall, is walking 1 m/s down a street at night toward a lightpost that stands 7 m tall. How fast is the length of the person's shadow changing when the person is 5 m away from the lightpost?



How fast does the tip of the shadow moving

$$(T = l + p)$$

$$\frac{dT}{dt} = \frac{dl}{dt} + \frac{dp}{dt}$$

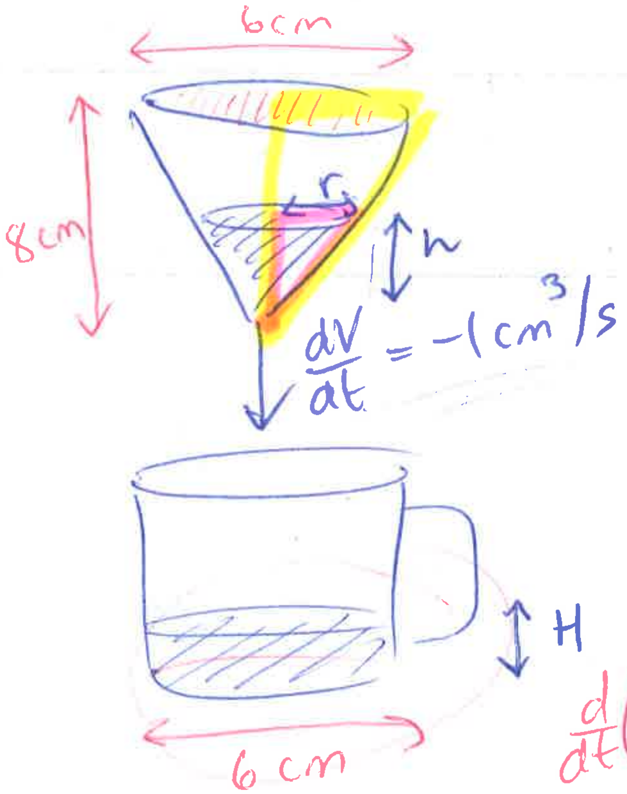
$$\left.\frac{dT}{dt}\right|_{p=5} = -1\text{ m/s} - 0.346\text{ m/s}$$

$$= -1.346\text{ m/s}$$

$$\frac{l+p}{7} = \frac{l}{1.8}$$

$$\frac{1}{7}\left(\frac{dl}{dt} + \frac{dp}{dt}\right) = \frac{1}{1.8}\frac{dl}{dt}$$

Example: Water is flowing out of a conical filter into a cylindrical cup at a constant rate of 1 cm³/sec. If the ~~filer~~ filter has a height of 8 cm and diameter of 6 cm and the cylindrical cup has the same diamter determine how fast the height of water in the filter and cup are changing.



$$\frac{dh}{dt} = ? \qquad \frac{dH}{dt} = ?$$

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

$$\frac{r}{h} = \frac{3}{8}$$

$$r = \frac{3}{8}h$$

$$\frac{d}{dt}\left(V(h) = \frac{1}{3}\pi\left(\frac{3}{8}h\right)^2 h\right)$$

$$\frac{3\pi}{64}h^3$$

$$\Rightarrow \frac{dV}{dt} = \frac{9\pi}{64}h^2\frac{dh}{dt} \Rightarrow \boxed{\frac{dh}{dt} = \frac{-64}{9\pi h^2}} \Rightarrow \text{as } h \to 0 \quad \left|\frac{dh}{dt}\right| \to \infty$$

$$V_{cyl.} = \pi r^2 \cdot H$$

$$\frac{d}{dt}\left(V = 9\pi H\right)$$

$$\Rightarrow \underbrace{\frac{dV}{dt}}_{+1} = 9\pi\frac{dH}{dt} \Rightarrow \boxed{\frac{dH}{dt} = \frac{1}{9\pi}} \Rightarrow \text{const.}$$

Practice Problems: 3.5: 2D # 1-3, 5, 8-15 3D # 4, 6, 7, 16, 17