Slopes of Tangent Lines Quiz

Name: ____________________ Date: October 7, 2019

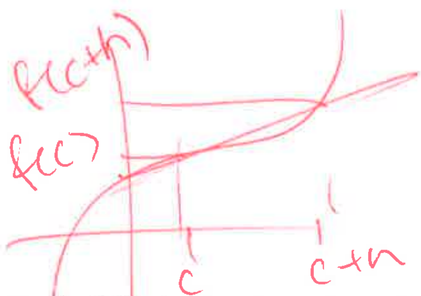
Thinking Strategies	Communication	Modelling & Solving

1. For a given function $f(x)$, we discused two similar ways of using limits to determine the slope at the point $x = c$. Explain how one of the two are defined.

$\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ $\frac{\Delta y}{\Delta x}$ OR $\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$ Δy

$c+h-c$
Δx

Both are $\frac{\Delta y}{\Delta x}$ = slope



slope is $\frac{f(c+h) - f(c)}{c+h-c}$ as $h \to 0$

2. For the following functions, determine the slope of the tangent at $x = c$ and check at $c = 0$ using a table.

   (a) $g(x) = \frac{1}{x^2+1}$

$\lim_{h \to 0} \frac{\frac{1}{(c+h)^2+1} - \frac{1}{c^2+1}}{h}$

@ $x = c = 0$
slope $\Rightarrow 0$

$= \lim_{h \to 0} \frac{\cancel{c^2}\cancel{+1} + (-\cancel{c^2} - 2c\cancel{h} - h^{\cancel{2}} \cancel{-1})}{\cancel{h}(c^2+1)((c+h)^2+1)}$

$= \lim_{h \to 0} \frac{-2c - h^{\to 0}}{(c^2+1)((c+h)^2+1)} = \frac{-2c}{(c^2+1)^2}$

near $x=0$	$\frac{\Delta y}{\Delta x}$
0.01	$\frac{\frac{1}{0.01^2+1} - 1}{0.01} = -0.01$
0.0001	-0.0001

to $\uparrow 0$ ✓

(b) $h(x) = \sqrt{x^2+1}$

$$\lim_{x \to c} \frac{\sqrt{x^2+1} - \sqrt{c^2+1}}{x-c} \cdot \frac{(\sqrt{x^2+1} + \sqrt{c^2+1})}{''}$$

$$\lim_{x \to c} \frac{x^2 + \cancel{1} - c^2 - \cancel{1}}{(x-c)(\sqrt{x^2+1} + \sqrt{c^2+1})}$$

$$\lim_{x \to c} \frac{x+c}{\sqrt{x^2+1} + \sqrt{c^2+1}} = \frac{\cancel{2}c}{\cancel{2}\sqrt{c^2+1}}$$

near $x=0$	$\frac{\Delta y}{\Delta x}$
0.01	$\frac{\sqrt{0.01^2+1} - 1}{0.01}$ $= 0.005$
0.0001	$= 0.00005$

↓ to 0 ✓

Year	2011	2012	2013	2014	2015	2016	2017	2018
Number of Monthly Twitter Users (millions)	117	185	241	288	305	318	330	321

3. See attached sheet for a graph of the above data regarding the number of monthly Twitter users from 2011 to 2018 (source: https://www.statista.com/statistics/282087/number-of-monthly-active-twitter-users).

   Determine the average rate of growth on Twitter in 2014. Justify what you believe to be the instantaneous rate of growth in 2014.

(1) on [2013, 2014] $\frac{\Delta y}{\Delta x} = 288 - 241 = 47$ million/year

(2) on [2014, 2015] $\frac{\Delta y}{\Delta x} = 305 - 288 = 17$ million/year

avg of (1) and (2) $= \frac{47+17}{2} = \boxed{32 \text{ million/year}}$

(3) on [2013, 2015]