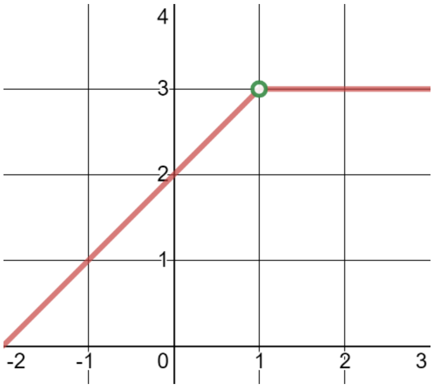
The Limit

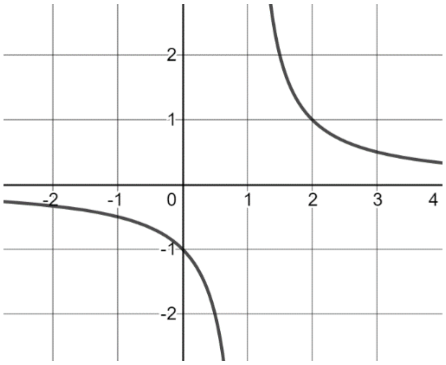
Goal: • Can describe a limit as a shrinking ball. • Can determine a limit from a graph and using a table of values. • Can determine a limit algebraically
Terminology: • Limit

Using the graphs and table of values. Describe the behaviour around $x = 1$.

A.



B.



C.

x	$f(x)$
2	5
1.1	7.1
1.05	7.68
1.0001	7.9648
1	9
0.9999	9.0431
0.95	9.14
0.9	9.6
0	14

Your group's description		
Class description		

What you are describing is the basic definition of the limit. That is:

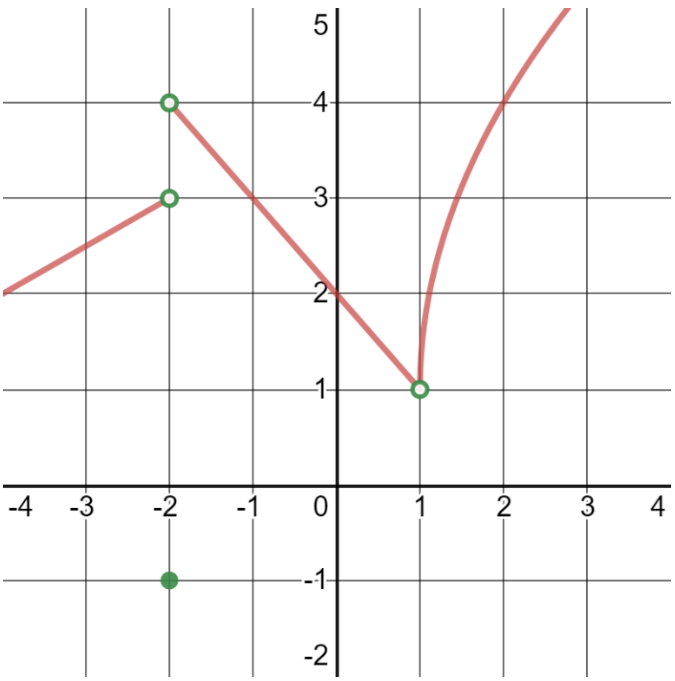
And we denote it as:

When we use the graph to determine a limit, we can imagine a ball centered around the limit point that is getting smaller and smaller around the limit point but still includes other parts of the function.

$$\lim_{x \to c} f(x) = L$$

If we can make a ball arbitrarily small around L and completely contain the function, we say the limit exists. However, if we can shrink the ball so it contains no other parts of the function (on either side), then the limit does not exist as this would mean the function does not behave regularly.

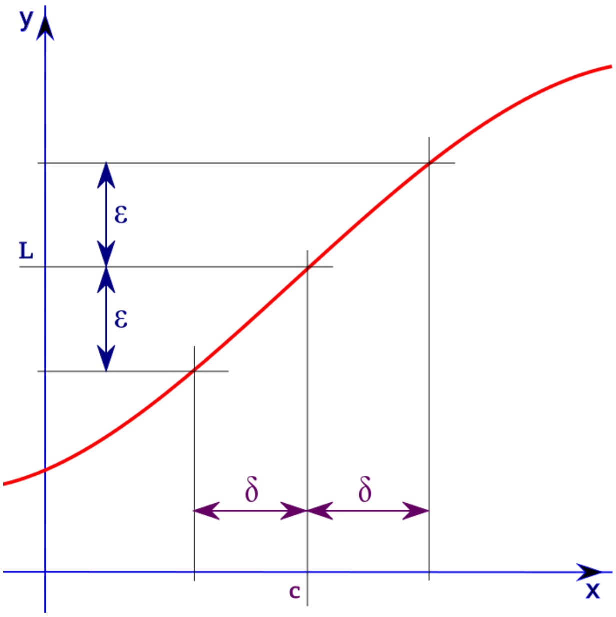
Example: Determine $\lim_{x \to 2} f(x)$ given f below

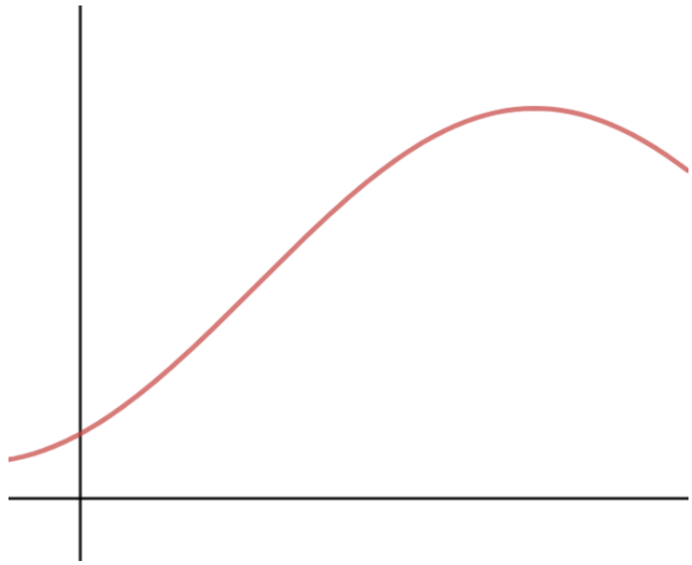


Practice: Determine $\lim_{x \to 1} f(x)$ for the same function above.

Practice: Determine $\lim_{x \to -2} f(x)$.

Another way of saying this, is that someone could say how close they want the function to be to L and we can find a small interval around c so that the function behaves within the desired tolerance.





If we can't see the limit from a graph, we can use a table of values just as we did last class to find the slope of the tangent line.

Example: Determine

$$\lim_{x \to 1} \frac{x - 1}{\sqrt{x} - 1}$$

Practice: Determine the following using a table and by using algebraic simplifications

$$\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 - 9}$$

Determining the value to a limit is not too bad if we remember that we have multiple ways (graphs, table of values, and algebraic simplification) to evaluate them. Since the limit is the behaviour around the point you could think of the limit as the approximation.

$$\lim_{x \to c} f(x) \approx f(c)$$

If they are equal, then we say the function is **continuous** and it can be drawn without lifting the pencil.

**Most of our discontinuities are from:

This makes the limit of some questions very trivial; however, the point of practicing these simple problems is that limits behave a lot like regular numbers and there are operations we can do with numbers that we can do with limits.

Assume that $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$ are real numbers, then:

$$\lim_{x \to c} \big(f(x) \pm g(x)\big) = L + M$$

$$\lim_{x \to c} \big(f(x) \cdot g(x)\big) = L \cdot M$$

$$\lim_{x \to c} \left(\frac{f(x)}{g(x)}\right) = \frac{L}{M}, \quad M \neq 0$$

$$\lim_{x \to c} k \cdot f(x) = k \cdot L$$

$$\lim_{x \to c} \big(f(x)\big)^n = L^n, \quad n \in \mathbb{Q}$$

Practice Problems: 1.2: # 1-6*, 7, 12

11, 13, Problems Plus #1