Velocity and Acceleration

Goal:
• Can use derivative rules to apply the rate of change of distance over time as velocity. • Can use derivative rules to apply the rate of change of velocity over time as acceleration. • Can analyze 1-dimensional movement of a particle on a line.
Terminology: • Velocity • Acceleration

We know that average velocity is $\frac{\Delta x}{\Delta t}$ where x is the distance and t is the time. We talked about average and instantaneous rate change as the slope of a distance vs time graph. In Physics 11 you get the following kinematic equations. Derive them and give them a calculus definition.

Physics 11 Definition	Calculus Definition and Explanation
$\Delta x = \left(\frac{v_1 + v_0}{2}\right) \cdot \Delta t$	calculus $\lim_{\Delta x, \Delta t \to 0}$ $\frac{\Delta x}{\Delta t}$ = avg vel. $\rightarrow$ $\frac{dx}{dt}$ = instantaneous veloc.
$v_1 = v_0 + a\Delta t$	$V_1 - V_0 = a\Delta t$ $\Delta v = a\Delta t$ $\rightarrow$ $\frac{dv}{dt}$ = instantaneous acceleration $\frac{\Delta V}{\Delta t} = a$ ★ avg. acc. $\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$
$\Delta x = v_0\Delta t + \frac{1}{2}a \cdot (\Delta t)^2$	$\frac{\Delta x}{\Delta t} = V_0 + \frac{1}{2}a\Delta t$ $\frac{dx}{dt} = V_0 + \frac{1}{2}a \cdot dt = V_0 + \frac{1}{2}dv$ initial velocity overall change in velocity $\rightarrow v(t)$ ★ Spring break
$v_1^2 = v_0^2 + 2a\Delta x$	ELFS

I want to briefly talk about motion along a line as it is a classic problem

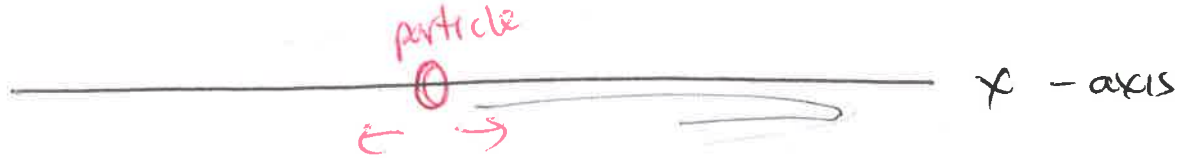
Example: Consider a particle moving along the x-axis given by the equation

$$x(t) = t^3 - 25t^2 + 171t - 315$$

time

Where $x(t)$ is the distance in meters after t minutes. Determine:

a. The velocity of the particle
b. The acceleration of the particle
c. Identify when the particle is moving to the left and right.
d. The total distance travelled from $t = 0$ to $t = 10$



a.) $v(t) = \frac{dx}{dt} = 3t^2 - 50t + 171$

b.) $a(t) = \frac{dv}{dt} = 6t - 50$

c.) when is particle moving to the right?

velocity is the speed/direction we move

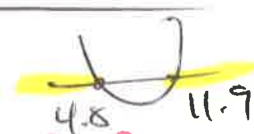
$v(t) > 0 \Rightarrow 3t^2 - 50t + 171 > 0$

$t = \frac{50 \pm \sqrt{2500 - 4(3)(171)}}{6}$

$= 11.9, 4.8$

$t > 11.9$ min

or $t < 4.8$ min



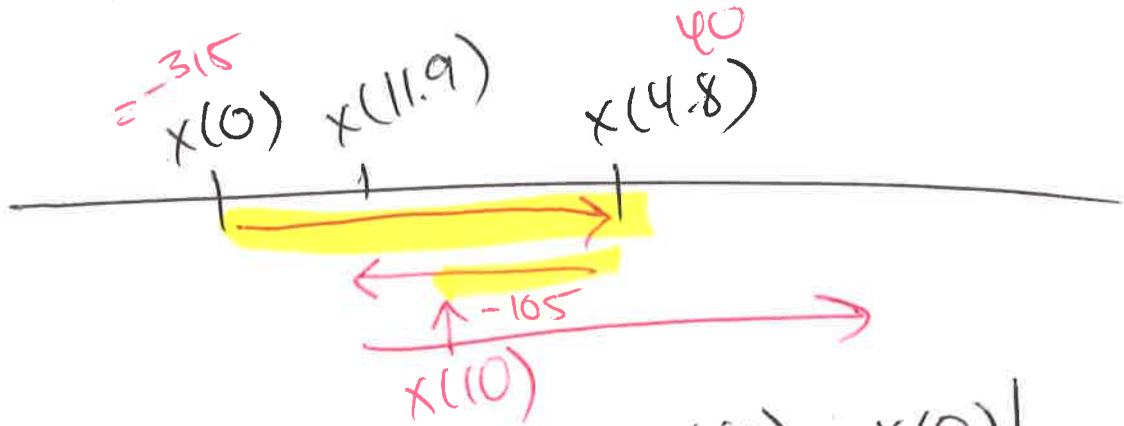
when is particle moving to the left?

$v(t) < 0 \Rightarrow 4.8 < t < 11.9$

d.) Total distance: $x(10) - x(0)$ total displacement

Practice Problems: 3.1: # 1, 2-6 (do what you need), 7-9
3.2: # 1-2, 3-4 (do what you need), 5-9
Give calculus meaning to the kinematic equations

Total distance



Total distance : $|x(4.8) - x(0)|$

$+ |x(10) - x(4.8)|$

$= |40.392 - (-315)|$

$+ |-105 - 40.392|$

$\sim \cancel{4}500$ m

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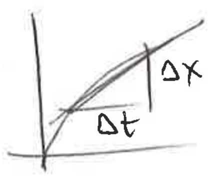
$\Delta x = \left(\frac{v_1 + v_0}{2}\right) \cdot \Delta t$	dist = avg vel. × time  $\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = v$ → instantaneous velocity
$v_1 = v_0 + a\Delta t$	$v_1 - v_0 = a\Delta t$ $\Delta v = a\Delta t$ $\frac{\Delta v}{\Delta t} = a_{avg.}$ → $\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = a_{inst.}$
$\Delta x = v_0\Delta t + \frac{1}{2}a \cdot (\Delta t)^2$	$\frac{\Delta x}{\Delta t} = v_0 + \frac{1}{2}a\Delta t$ initial velocity → $\frac{dx}{dt} = v_0 + \frac{1}{2}a\,dt$ → change in velocity spring break ⇒ $v(t)$  $a = \frac{dv}{dt}$ $a\,dt = dv$
$v_1^2 = v_0^2 + 2a\Delta x$	★ ELFS

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Example: Consider a particle moving along the x-axis given by the equation

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Where $x(t)$ is the distance in meters after t minutes. Determine:

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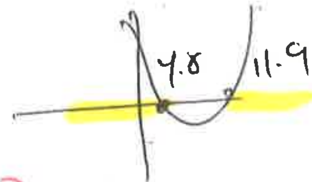
a.) $v(t) = \frac{dx}{dt} = 3t^2 - 50t + 171$



b.) $a(t) = \frac{dv}{dt} = 6t - 50$

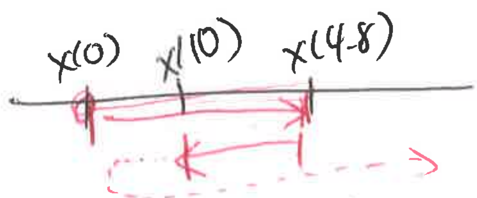
c.) ☆ when does the particle move right? $v(t) > 0$! (Direction is +ve!) $\Rightarrow 3t^2 - 50t + 171 > 0$

$$t = \frac{50 \pm \sqrt{2500 - 4(3)(171)}}{6} = 11.9,\ 4.8$$

$$\Rightarrow \boxed{t > 11.9\text{ min or } t < 4.8\text{ min}}$$

~~☆ when does~~ the particle move left? $v(t) < 0$

$$\boxed{4.8 < t < 11.9\text{ min}}$$



d.) $\underbrace{x(10)}_{\text{end}} - \underbrace{x(0)}_{\text{start}}$: displacement

Tot. dist $= |x(4.8) - x(0)| + |x(10) - x(4.8)|$
$|40.4 + 315| + |-105 - 40.4| = 500.8\text{ m}$

Practice Problems: 3.1: # 1, 2-6 (do what you need), 7-9
3.2: # 1-2, 3-4 (do what you need), 5-9
Give calculus meaning to the kinematic equations