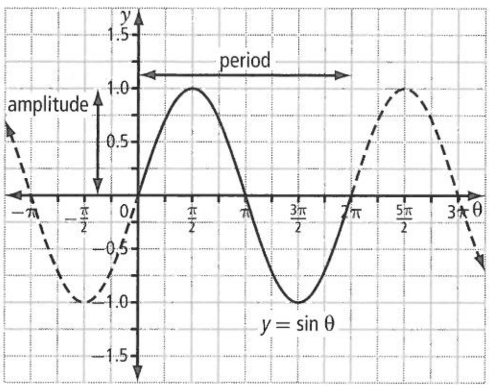
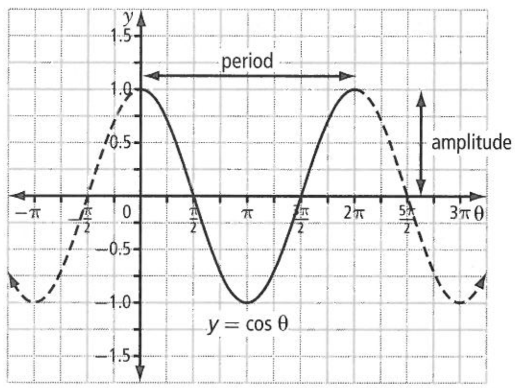
Chapter 5 Trigonometric Functions and Graphs

5.1 Graphing Sine and Cosine Functions

KEY IDEAS

- Sine and cosine functions are *periodic* or *sinusoidal functions*. The values of these functions repeat in a regular pattern. These functions are based on the unit circle.
- Consider the graphs of $y = \sin\theta$ and $y = \cos\theta$.





- The maximum value is +1.
- The minimum value is −1.
- The amplitude is 1.
- The period is 2π.
- The y-intercept is 0.
- The θ-intercepts on the given domain are $-\pi$, 0, π, 2π, and 3π.
- The domain of $y = \sin\theta$ is $\{\theta \mid \theta \in R\}$.
- The range of $y = \sin\theta$ is $\{y \mid -1 \le y \le 1, y \in R\}$.

- The maximum value is +1.
- The minimum value is −1.
- The amplitude is 1.
- The period is 2π.
- The y-intercept is 1.
- The θ-intercepts on the given domain are $-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$, and $\frac{5\pi}{2}$.
- The domain of $y = \cos\theta$ is $\{\theta \mid \theta \in R\}$.
- The range of $y = \cos\theta$ is $\{y \mid -1 \le y \le 1, y \in R\}$.

- For sinusoidal functions of the form $y = a \sin bx$ or $y = a \cos bx$, a represents a vertical stretch of factor $|a|$ and b represents a horizontal stretch of factor $\frac{1}{|b|}$. Use the following key features to sketch the graph of a sinusoidal function.
  - the maximum and minimum values
  - the amplitude, which is one half the total height of the function

    $$\text{Amplitude} = \frac{\text{maximum value} - \text{minimum value}}{2}$$

    The amplitude is given by $|a|$.
  - the period, which is the horizontal length of one cycle on the graph of a function

    $$\text{Period} = \frac{2\pi}{|b|} \text{ or } \frac{360°}{|b|}$$

    Changing the value of b changes the period of the function.
  - the coordinates of the horizontal intercepts

Working Example 1: Graph the Sine and Cosine Functions

Graph each function for the domain $0 \leq \theta \leq 3\pi$.

a) $y = \sin\theta$ **b)** $y = \cos\theta$

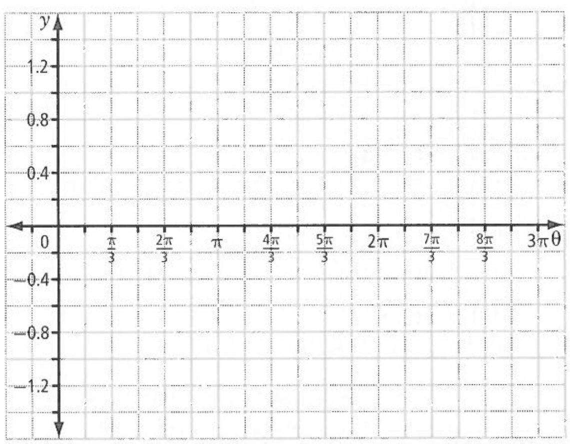
Solution

a) Complete the table of values. Round values to three decimal places.

θ	$y = \sin\theta$	θ	$y = \sin\theta$	θ	$y = \sin\theta$	θ	$y = \sin\theta$
0		$\frac{6\pi}{6} = \pi$		$\frac{12\pi}{6} = 2\pi$		$\frac{18\pi}{6} = 3\pi$	
$\frac{\pi}{6}$		$\frac{7\pi}{6}$		$\frac{13\pi}{6}$			
$\frac{2\pi}{6} = \frac{\pi}{3}$		$\frac{8\pi}{6} = \frac{4\pi}{3}$					
		$\frac{9\pi}{6} = \frac{3\pi}{2}$		$\frac{15\pi}{6} = \frac{5\pi}{2}$			
$\frac{4\pi}{6} = \frac{2\pi}{3}$				$\frac{16\pi}{6} = \frac{8\pi}{3}$			
$\frac{5\pi}{6}$							

Make sure your calculator is in radian mode.

Plot the points and join them with a smooth curve.

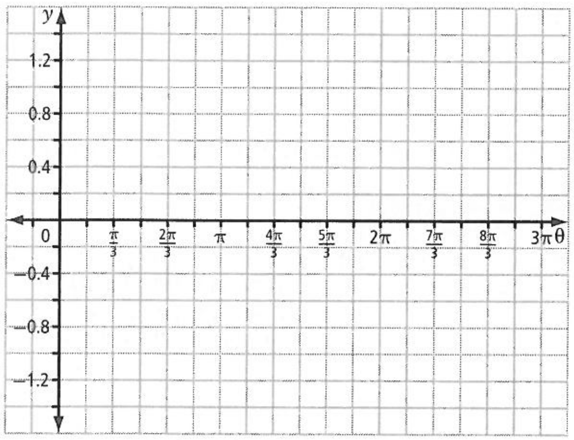


b) Complete the table of values.

θ	$y = \cos\theta$	θ	$y = \cos\theta$	θ	$y = \cos\theta$	θ	$y = \cos\theta$
0		π		2π		3π	
$\frac{\pi}{6}$		$\frac{7\pi}{6}$					
$\frac{\pi}{3}$		$\frac{4\pi}{3}$					
$\frac{\pi}{2}$		$\frac{3\pi}{2}$					
$\frac{2\pi}{3}$		$\frac{5\pi}{3}$					
$\frac{5\pi}{6}$		$\frac{11\pi}{6}$					

What will the value for 2π be? Which angle is coterminal with 2π?

Plot the points and join them with a smooth curve.

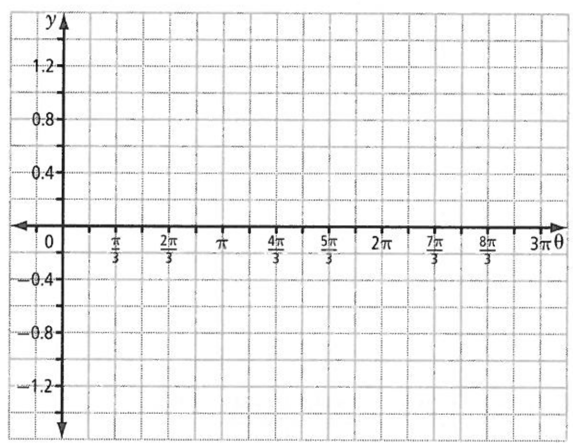


Working Example 2: Determine the Period of a Sine Function

Graph $y = \sin(3\theta)$ on the domain $0 \leq \theta \leq 3\pi$.

Solution

First consider the key features of the basic sine function, $y = \sin \theta$.
Use the key features to sketch the graph of the basic sine function, $y = \sin \theta$.



Now consider the function $y = \sin(3\theta)$.

- The amplitude of $y = \sin(3\theta)$ is ________.
- The maximum value of $y = \sin(3\theta)$ is ________.
- The minimum value of $y = \sin(3\theta)$ is ________.
- The period of $y = \sin(3\theta)$ is $\frac{2\pi}{|b|}$, or ________.
- The θ-intercepts of $y = \sin(3\theta)$ on the domain $0 \leq \theta \leq 3\pi$ are ________________.
- The y-intercept of $y = \sin(3\theta)$ is ________.

> How can you use the period of $y = \sin(3\theta)$ and the θ-intercepts of the basic sine function to determine the θ-intercepts of $y = \sin(3\theta)$?

Use the key features to sketch the graph of the function $y = \sin(3\theta)$ on the coordinate grid above.

Working Example 3: Determine the Amplitude of a Cosine Function

Graph $y = -4 \cos \theta$ on the domain $0 \leq \theta \leq 3\pi$.

Solution

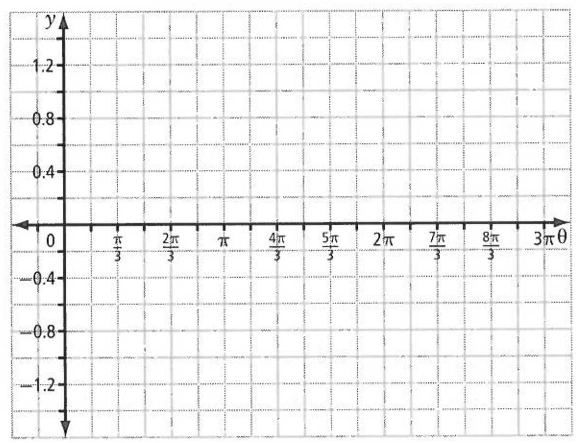
Complete the table. First, determine the key features of $y = \cos \theta$ on the specified domain.

	$y = \cos \theta$	$y = -4 \cos \theta$
Maximum value		
Minimum value		
Amplitude		
Period		
θ-intercepts		
y-intercept		

How does the amplitude of $y = -4 \cos \theta$ compare to the amplitude of $y = \cos \theta$?

How do the θ-intercepts of $y = -4 \cos \theta$ compare to the θ-intercepts of $y = \cos \theta$?

Use the key features to sketch the graph of the basic cosine function, $y = \cos \theta$. Then, sketch the graph of $y = -4 \cos \theta$ on the same coordinate grid.



Check Your Understanding

Practise

1. State the amplitude of each trigonometric function.

a) $y = 2 \cos \theta$

b) $y = \frac{1}{4} \sin \theta$

c) $y = 5 \sin (2\theta)$

d) $y = -3 \cos \left(\frac{1}{2}\theta\right)$

2. State the period of each trigonometric function in degrees and in radians.

a) $y = 3 \sin \theta$

Degrees: $\frac{360°}{|b|}$ = ________ Radians: $\frac{2\pi}{|b|}$ = ________

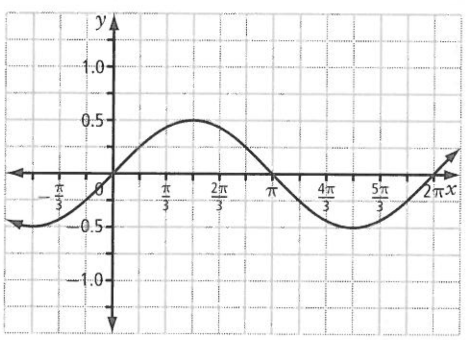
b) $y = \cos (2\theta)$

c) $y = 0.25 \sin (0.25\theta)$

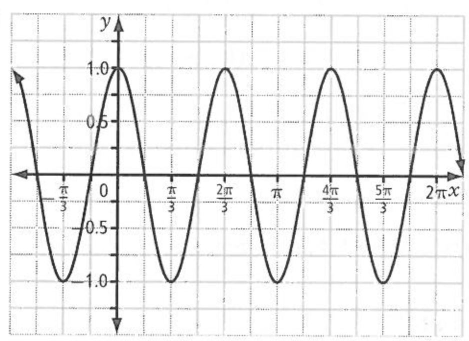
d) $y = -1.5 \cos (1.5\theta)$

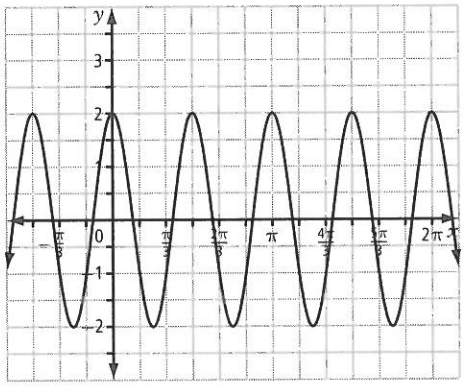
3. State the period, in radians, and the amplitude of each trigonometric function.

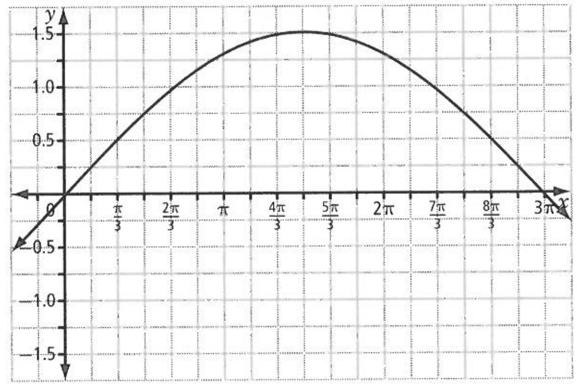
a)



b)



c) 

d) 

4. Identify the key features of $y = \sin \theta$ and the transformed sine function. Then, graph at least two cycles of the transformed sine function.

a) $y = \sin\left(\frac{1}{3}\theta\right)$

Identify the key features of $y = \sin \theta$.

$a =$ ______; the amplitude is ______.

Maximum value: ______ Minimum value: ______

$b =$ ______; the period is ______.

θ-intercepts: ____________ y-intercept: ______

Identify the key features of $y = \sin\left(\frac{1}{3}\theta\right)$.

$a =$ ______; the amplitude is ______.

The graph ______ reflected in the x-axis.
(*is* or *is not*)

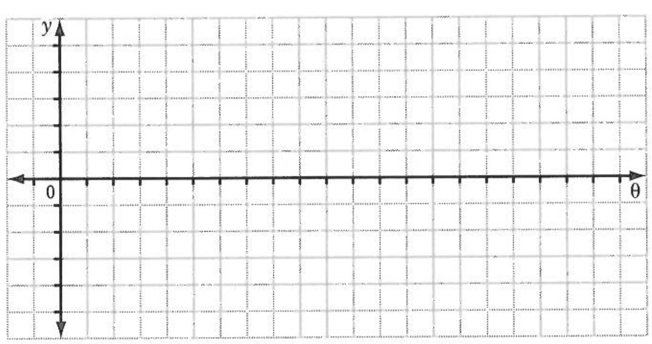
Maximum value: ______ Minimum value: ______

$b =$ ______; the period is ______.

The graph is stretched ____________ by a factor of ______.
(*horizontally* or *vertically*)

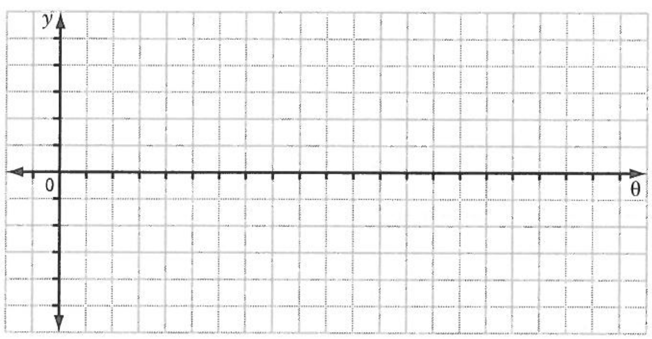
θ-intercepts: ____________ y-intercept: ______

Use the key features to sketch the graph of the function.

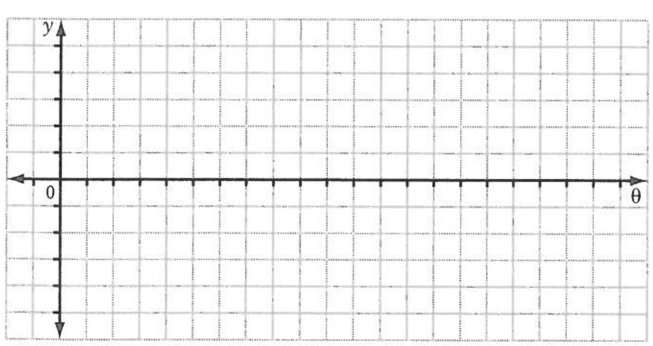


Consider the key features of the function when choosing the scales.

b) $y = 1.5 \sin (2\theta)$

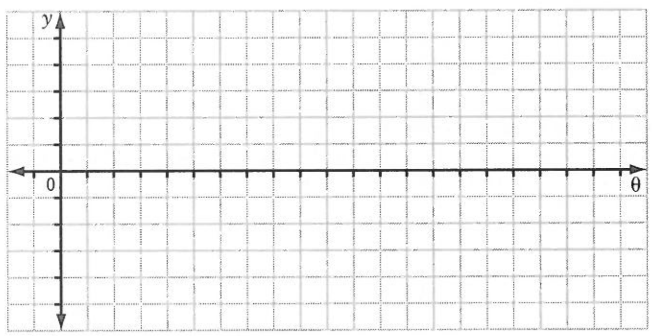


c) $y = -2 \sin (4\theta)$

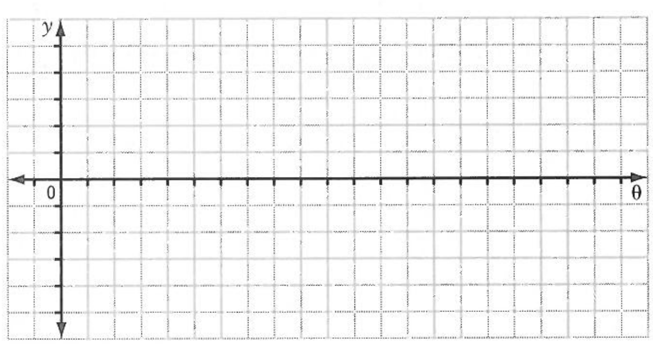


5. Identify the key features of each cosine function. Then, graph at least two cycles of each cosine function.

a) $y = 2\cos\left(\frac{1}{2}\theta\right)$



b) $y = -\cos(2\theta)$



Connect

6. Consider the graphs of $y = \sin x$ and $y = \cos x$ below.

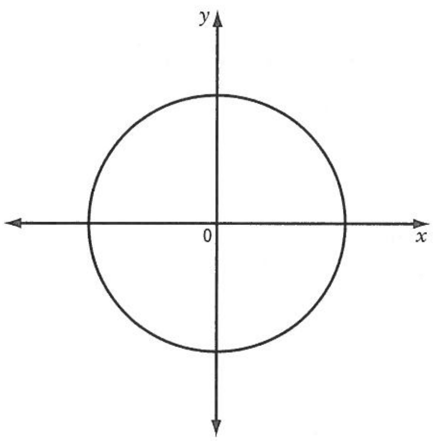
a) Divide the graph of each trigonometric function into quadrants. Label the quadrants on the graphs and on the unit circles.

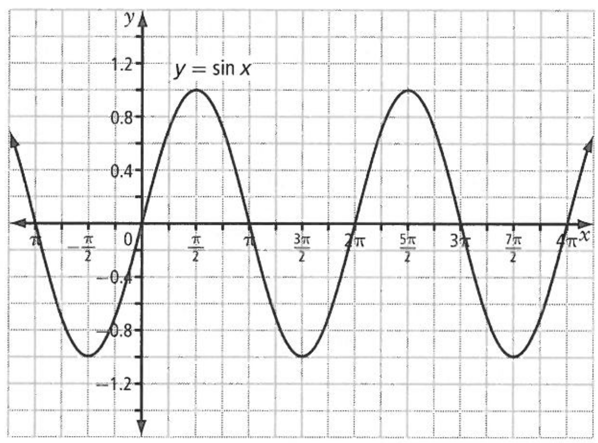
b) Shade in the regions where $\sin x$ or $\cos x$ is positive on each graph and on each unit circle. Identify the maximum values.

c) Using a different colour, shade in the regions where $\sin x$ or $\cos x$ is negative on each graph and on each unit circle. Identify the minimum values.

d) Identify the x-intercepts and the corresponding angles on the unit circle.

Sine Function





Cosine Function