5.4 Equations and Graphs of Trigonometric Functions

KEY IDEAS

- You can use sinusoidal functions to model periodic phenomena that do not involve angles as the independent variable. Examples of such phenomena include
  – wave shapes, such as a heartbeat or ocean waves
  – pistons in a machine or the swing of a pendulum
  – circular motion, such as a Ferris wheel
- You can adjust the parameters *a*, *b*, *c*, and *d* in sinusoidal equations of the form $y = a \sin b(\theta - c) + d$ or $y = a \cos b(\theta - c) + d$ to fit the characteristics of the phenomenon being modelled.
- Graphing technology allows you examine how well the model represents the data. It also allows you to extrapolate or interpolate solutions from the model.
- You can find approximate solutions to trigonometric equations using the graphs of the trigonometric functions. Express solutions over a specific interval or give a general solution.

Working Example 1: Solve Simple Trigonometric Equations

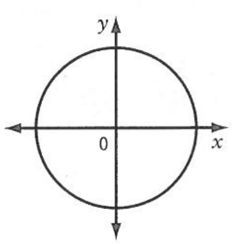
Solve each equation over the specified interval.

a) $\sin x = 0.5, 0° \leq x \leq 720°$

b) $\sin 2x = 0.5, 0° \leq x \leq 720°$

Solution

a) Method 1: Use the Unit Circle and Special Triangles

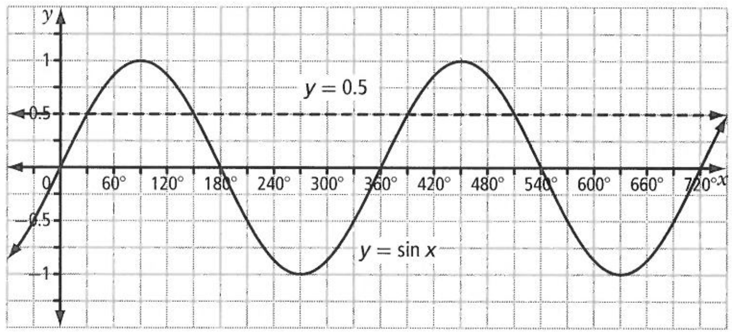


$\theta_R = \sin^{-1}(0.5) =$ ________

The solutions are $x =$ ________, ________, ________, ________, $0° \leq x \leq 720°$.

Method 2: Use a Graph

Graph $y = \sin x$ (Left Side) and $y = 0.5$ (Right Side) on the same set of axes on the domain $0° \leq x \leq 720°$.

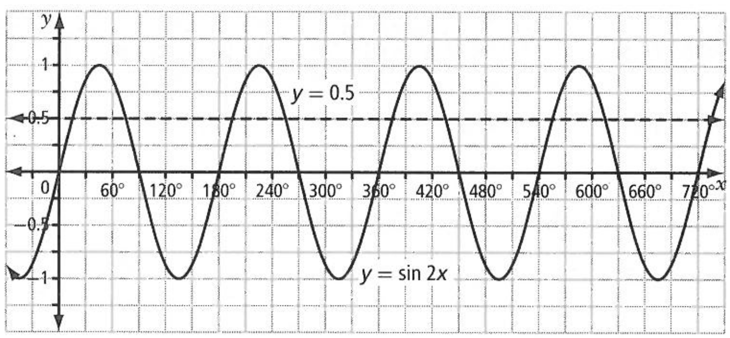


Check that your calculator is set to the correct mode.

Determine the coordinates of the points of intersection of the two functions.

The solutions are $x =$ ________, ________, ________, ________, $0° \leq x < 720°$.

b) Graph $y = \sin 2x$ (Left Side) and $y = 0.5$ (Right Side) on the same set of axes over the domain $0° \leq x \leq 720°$.



How many solutions are there?

The solutions are $x =$ ____, $0° \leq x < 720°$.

Working Example 2: Solve a Trigonometric Equation by Graphing

Solve $3 = 16 \cos [5(x + 1)] - 5$, $0 \leq x \leq \pi$.

Solution

Collect all terms on the same side of the equation.

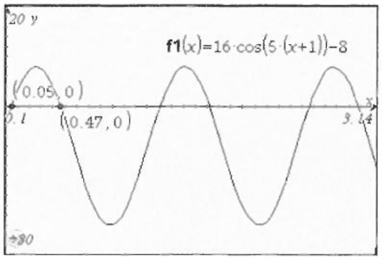
$3 = 16 \cos [5(x + 1)] - 5$

$0 =$ ____

Substitute y for 0.

$y =$ ____

Graph the related function using graphing technology.



How many solutions are there within the given domain?

The solutions within the first cycle are $x_1 \approx$ ________ and $x_2 \approx$ ________.

Additional solutions are of the form $x + n$ (period).

The solutions are $x \approx$ ____, $0 \leq x \leq \pi$.

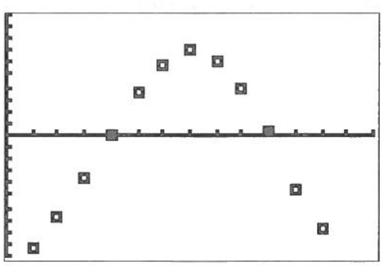
Working Example 3: Model Average Temperature

Write a sinusoidal function that models the average temperature each month in Whitehorse, Yukon, in a given year.

Month	Temperature (°C)	Month	Temperature (°C)	Month	Temperature (°C)
1	−18.4	5	6.8	9	7.5
2	−13.4	6	11.9	10	0.6
3	−7.3	7	14.0	11	−9.1
4	0.1	8	12.3	12	−15.7

Solution

Use graphing technology to create a scatter plot of the data.



Let x represent ______________________. Let y represent ______________________.

Use a cosine model, $y = a \cos b(x - c) + d$, where $a < 0$.

$$|a| = \frac{\text{maximum value} - \text{minimum value}}{2}$$

$=$

Period: ________

Use the period to determine b, where $\frac{2\pi}{|b|}$ = period.

Determine the phase shift.

At what x-value does the minimum of $y = -\cos x$ occur? At what x-value does the minimum of the table data occur?

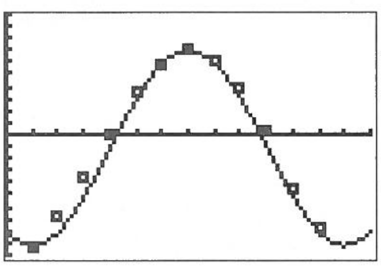
$c =$ ________

$$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$

$=$

Write your equation: ____

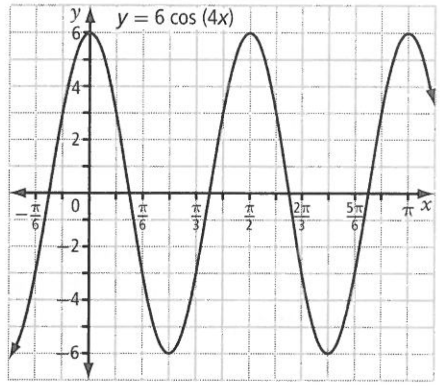
Use graphing technology. Graph the equation with the scatter plot to check that it is a reasonable representation of the data.



Check Your Understanding

Practise

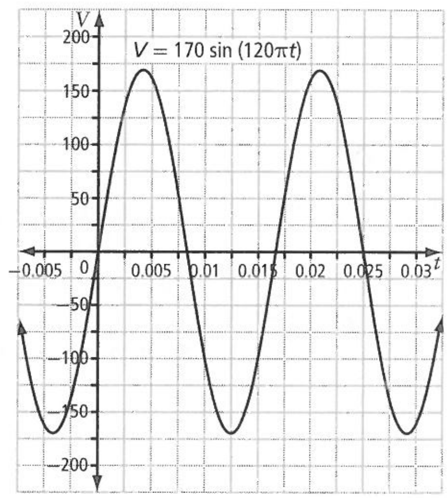
1. Use the graph of $y = 6 \cos (4x)$ to solve each trigonometric equation.



a) $6 \cos (4x) = 3, 0 \leq x \leq \pi$

b) $6 \cos (4x) = -6$, general solution in radians

2. Use the graph of $V = 170 \sin (120\pi t)$ to approximate the solutions to each trigonometric equation.



a) $50 = 170 \sin (120\pi t), 0 \leq x \leq 0.030$

b) $170 \sin (120\pi t) = -120, 0 \leq x \leq 0.030$

c) $170 \sin (120\pi t) = 0$, general solution in radians

3. Sound travels in waves. You can see the sinusoidal patterns of sound waves using a device called an oscilloscope.

a) Orchestra members tune their instruments to $A = 440$ Hz, meaning the sound wave repeats 440 times per second. What is the period of this sound wave, in seconds?

b) Write a simple sine function representing the waveform of the note $A = 440$.

$\frac{2\pi}{|b|} = \text{period}$

c) "Middle C" has a frequency of 261.63 Hz. What sine function could represent middle C?

4. Electricity comes into your home or school as alternating current, which can be modelled by a sinusoidal function. Electrical devices operate at the root mean square voltage, which is $\frac{1}{\sqrt{2}}$ of the peak voltage.

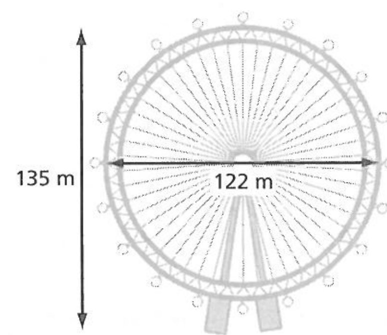
a) In Canada, many electrical devices require 120 V and 60 Hz. Write a sine function that represents the peak voltage in Canada.

To get the amplitude of the wave, multiply the required voltage by $\sqrt{2}$ ($\approx$1.4).

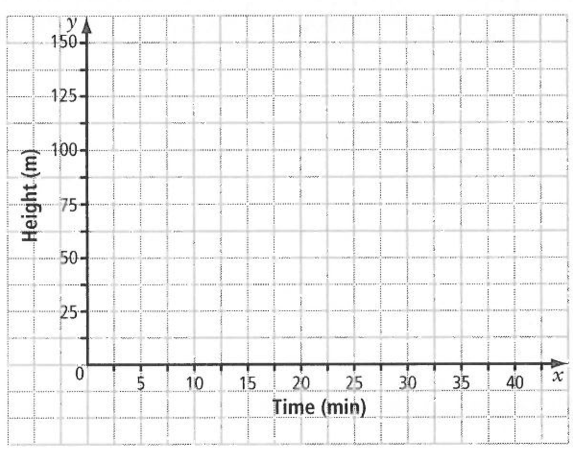
b) In Europe (and in most of Asia, Africa, and parts of South America), many appliances require 220 V and 50 Hz. Write a sine function that represents the peak voltage in Europe.

Apply

5. The London Eye has diameter 122 m and height 135 m. It takes approximately 30 min for one rotation of the wheel. Passengers board at the bottom of the ride. The ride moves slowly enough that it is usually not necessary for the wheel to stop to let passengers on or off.



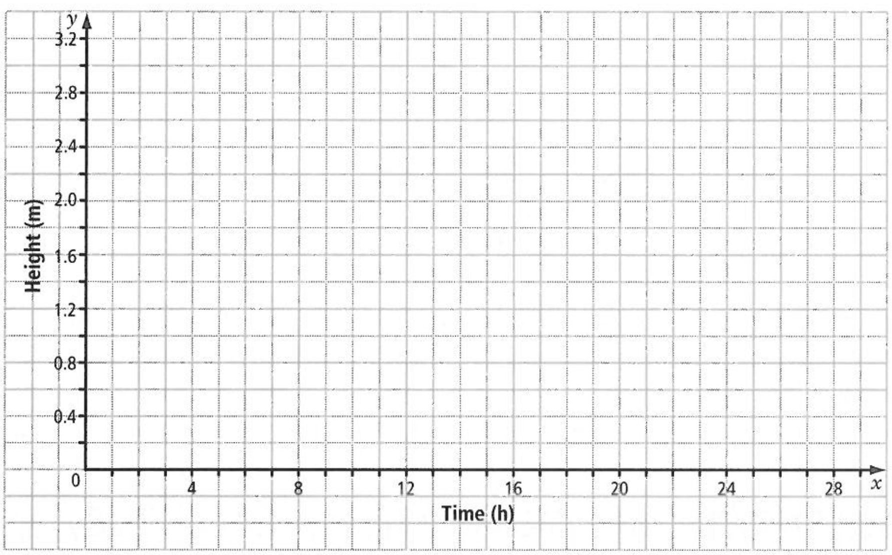
a) Sketch a sinusoidal function representing the height of a passenger riding the London Eye. What assumptions do you have to make?



b) Write a sinusoidal function that represents the height of a passenger riding the London Eye. Over what domain is the function valid?

6. One particular afternoon, the tide in Victoria, BC, reached a maximum height of 3.0 m at 2:00 p.m. and a minimum height of 0.2 m at 8:00 p.m.

a) Sketch a sinusoidal function based on these data. What assumptions do you have to make?



b) Write a sinusoidal function that represents the tide in Victoria, BC, on this day. Over what domain is the function valid?

7. Write a sinusoidal function that models the average temperature in Brandon, Manitoba. Use graphing technology to verify that your function is a good representation of the data.

	Jan 1	Feb 2	Mar 3	Apr 4	May 5	Jun 6	Jul 7	Aug 8	Sep 9	Oct 10	Nov 11	Dec 12
°C	−18.3	−15.8	−7.9	3.5	10.8	16.0	18.9	17.4	11.8	5.1	−5.3	−13.7

Connect

8. Using examples from class, from your textbook, and from this workbook, brainstorm a list of situations that can be modelled using a sinusoidal function in the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$. Next to each item, list any helpful information for constructing the model. One example has been provided to help you get started.

Situation	Notes
Circular motion	• $\lvert a \rvert$ = radius of the circle