Review

Chapter 1 Review

TRANSFORMATIONS

To transform a function, f, we can stretch it or compress it in the vertical or horizontal directions, reflect it over either primary axis, and shift it in the vertical or horizontal directions.

In function notation these transformations look like:

$$a \cdot f\left(\frac{1}{b}(x-c)\right)+d$$

And the equivalent mapping notation looks like:

$$(x,y) \mapsto (bx+c, ay+d)$$

Where a, b are vertical and horizontal stretch factors respectively. If they are negative, there is a reflection over an axis. The vertical and horizontal shift amounts are given by c and k. Written in this manner expansions/compressions/reflections must be done before translations since we follow order of operations in the mapping notation.

*** **THIS DOES NOT MEAN IT IS IMPOSSIBLE TO SHIFT BEFORE WE STRETCH**!! If you see $f(cx+d)$ you can interpret this as a horizontal shift d units and then a stretch by c units OR put it in the standard form $f\left(c\left(x+\frac{d}{c}\right)\right)$ as a horizontal stretch by c units and then a shift by d/c units.

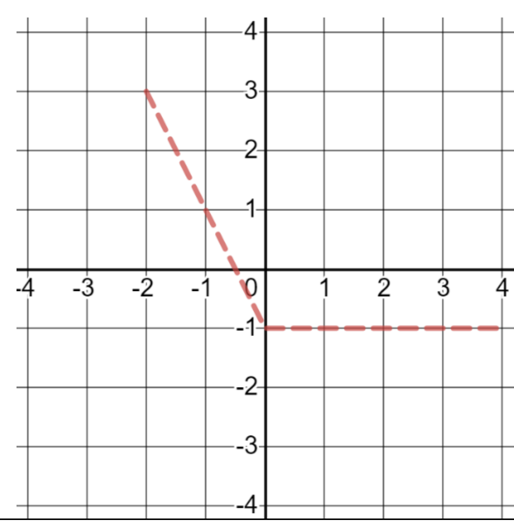
Note that mapping notation does exactly what it says horizontally and vertically. If it multiplies by a number with absolute value larger than 1, then it will be an expansion as the values gets larger. If it multiplies it by a small number with absolute value less than 1, then it will be a compression as the value gets smaller. Positive shifts are right and up and negative shifts are left and down.

Big Ideas you need to know and understand:

- Mapping notation does transformation is order of operation and nothing is reversed. Function notation reverses the horizontal transformations
- Can take either a set of verbal instruction, function notation, or mapping notation and create the other 2 from it and graph the image.
- To find a graph of the image function, we can either do each transformation step by step, or use mapping notation to see where coordinates get mapped to.

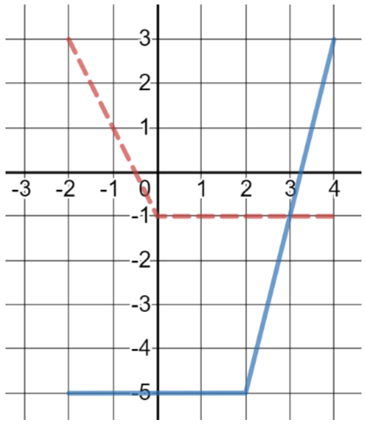
Review questions: Given the graph of $f(x)$ graph the image $g(x)$ and write the mapping notation and function notation of each transformation.

1. $g(x) = 2f(-x+2) - 3$
2. $T: (x,y) \mapsto \left(\frac{1}{2}x - 2, -y+1\right)$
3. Shift it down by 1, then compress vertically by 2. Reflect it over the y-axis

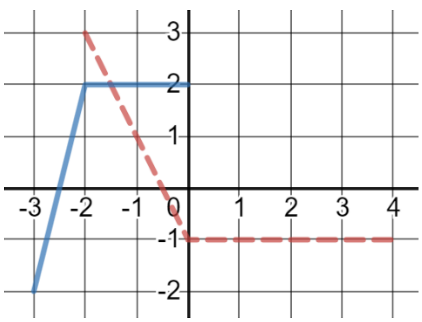


Solutions:

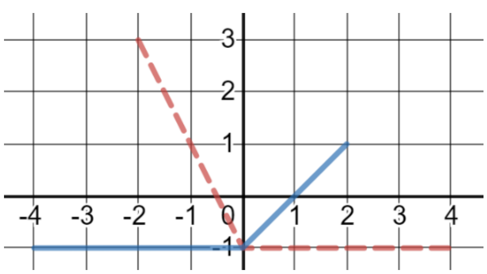
1. $T: \mathbb{R}^2 \to \mathbb{R}^2$
   $T: (x, y) \mapsto (-(x-2), 2y-3) = (-x+2, 2y-3)$



2. $g(x) = -f\big(2(x+2)\big) + 1$



3. $g(x) = \frac{1}{2}(f(-x) - 1)$ and $T: (x, y) \mapsto \left(-x, \frac{y-1}{2}\right)$ where $T: \mathbb{R}^2 \to \mathbb{R}^2$



INVERSES OF FUNCTIONS

When we invert a function, we go in the reverse order, that is from the range to the domain. If we have an ordered pair (x, y) it is implied that $x \mapsto y$ so the inverse would be $y \mapsto x$ and is the coordinate (y, x).

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To graph an inverse, simply graph the new (y, x) coordinates. This amounts to reflecting the graph over the line $y = x$ since all points in the form (a, a) will stay the same and be invariant.

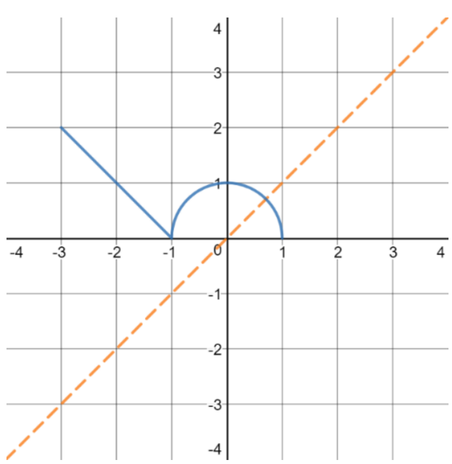
Reflect over the line $y = x$. Notice some key points that get mapped are:

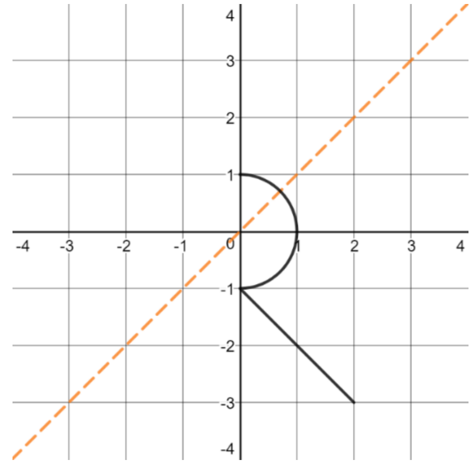
$(-3,2) \mapsto (2,-3)$

$(-1,0) \mapsto (0,-1)$

$(0,1) \mapsto (1,0)$

$(1,0) \mapsto (0,1)$





Some functions, like the one above, are not 1-to-1 (some outputs can be mapped to from different inputs and fails the horizontal line test) and as a result their inverses are not functions (the same input will yield different outputs and it fails the vertical line test). To describe such an inverse as a function, the domain of the original function needs to be restricted so it is 1-to-1.

Big Ideas you need to know and understand:

- All 1-to-1 functions have an inverse function. If not 1-to-1 just adjust the domain so that the function is 1-to-1 (passes the horizontal line test).
- Since the original function is just a series of instructions in order, the inverse is the same instructions in the reverse order and opposite operation so that everything is undone.

Review questions:

Find the inverse of the following functions

4. $f(x) = \sqrt{x+5} - 3$
5. $f(x) = \frac{x+4}{x+1}$
6. $f(x) = x^2 - 4$ that includes the point $x = 1$
7. $f(x) = -2(x-3)^2$ that includes the point $x = 1$

Solutions:

4. $f^{-1}(x) = (x+3)^2 - 5$
5. $f^{-1}(x) = \frac{x-4}{1-x}$
6. $f^{-1}(x) = \sqrt{x+4}$, we need that $x \geq 0$ in $f(x)$
7. $f^{-1}(x) = -\sqrt{-\frac{x}{2}} + 3$, we need that $x \leq 3$ in $f(x)$