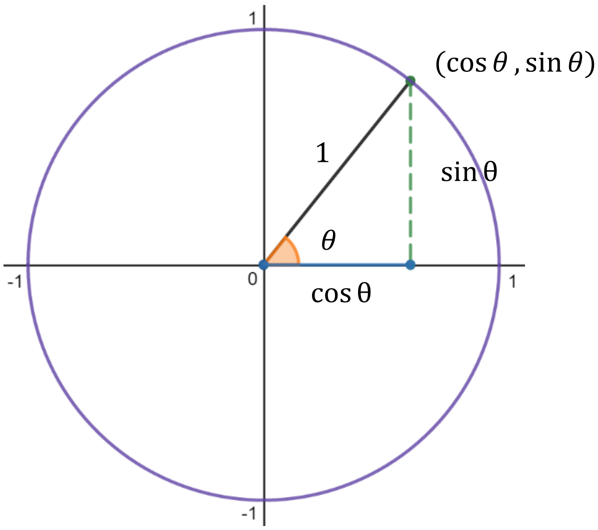
Chapter 5 Review

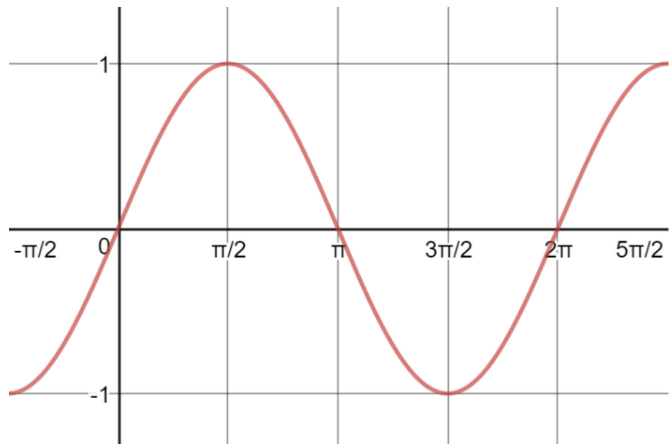
The Sine and Cosine Graph

We use the unit circle to graph sine and cosine. Sine is the y value and will start at 0 and make the following path:

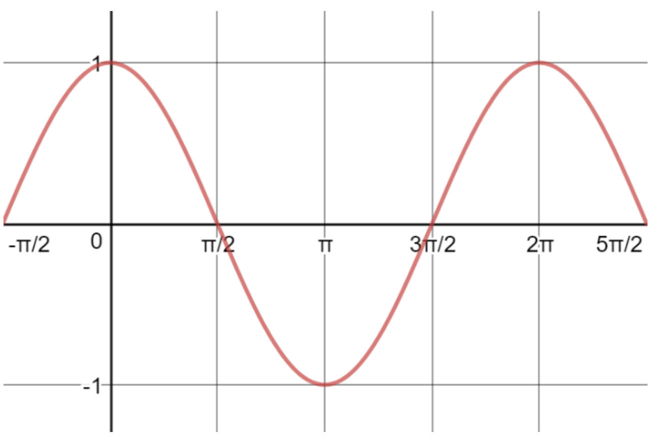
$$0 \to 1 \to 0 \to -1 \to 0$$

Which results in the graph





Cosine, on the other hand start at 1 and make the following path $1 \to 0 \to -1 \to 0 \to 1$



We can see that cosine is just a shift of sine, so their characteristics are virtually the same, just that starting point. The **amplitude** is half of the vertical distance from maximum to minimum. On sine and cosine, the amplitude is 1. The **period** is the horizontal distance it takes to repeat the pattern (from max to max, or min to min). Both sine and cosine have a period of 2π. The **midline** is the average value of the function. Sine and cosine have a midline at $y = 0$ (the x-axis).

There is *a lot* of symmetry in the graph of sine and cosine! Use the symmetry to help you make nice graphs!

Big Ideas you need to know and understand:
Memorize the shape of sine and cosine. That means knowing key points it passes through (zeros and extremum) and the basic characteristics of the graph (period, amplitude, midline, domain, range, etc.)
Cosine is just sine shifted by $90°$ or $\pi/2$.

Review questions:

1. Graph a full period of sine and cosine. Label key points.
2. Why do cosine and sine make the same shape?

Solutions:

1. See above graphs.
2. Both graphs are looking at the value of the x OR y coordinates on the unit circle. Since the unit circle is symmetric over each axis, the set of x points is the same as the set of y points they just have a different order.

TRANSFORMING THE SINE/COSINE GRAPH

We want to be able to graph and find the equation to trig functions of the form

$$a \cdot \sin\big(b(x-c)\big) + d$$

$|a|$: The **amplitude** of the trig function. If $a < 0$ then the graph will move in the opposite direction (ie sine goes down first, cosine goes up first).
b : The change in **period.** The new period is $2\pi/b$
c : The **phase shift**. How far left or right the graph is shifted.
d : The **vertical displacement**. How far up or down the midline was moved.

**Note that sine is odd so $\sin(-x) = -\sin x$ and cosine is even so $\cos(-x) = \cos x$ hence the value of b can always be made positive.

If we are given a graph or a set of points and asked to find the equation to it, we should start by finding the midline, then amplitude, then the period of our function.
** If we know the period, T, then we have $b = 2\pi/T$

Once we know the above, we can decide if we have a sine or cosine graph and then measure what the phase shift should be.

Example: Find two equations to the function that has a maximum at $(0,3)$ and nearest minimum is $(-7,-5)$

Solution: It might help to sketch it out as you go along. The midline is -1 as that is halfway between 3 and -5. The amplitude is 4 since it is four units from the midline to the max. The period is 14 since it takes half a period to go from -7 to 0.
Our equations should look like

$$\pm 4\sin\left(\frac{\pi}{7}(x-c)\right) - 1$$

$$\pm 4\cos\left(\frac{\pi}{7}(x-c)\right) - 1$$

To fill in the sign of the function and the value of c we need to understand the shape of our function. Cosine starts at the top, same as our function (no phase shift and positive), sine starts at the middle (-3.5 or 3.5 for our graph) and goes up and then down.

$$-4\sin\left(\frac{\pi}{7}(x-3.5)\right) - 1$$

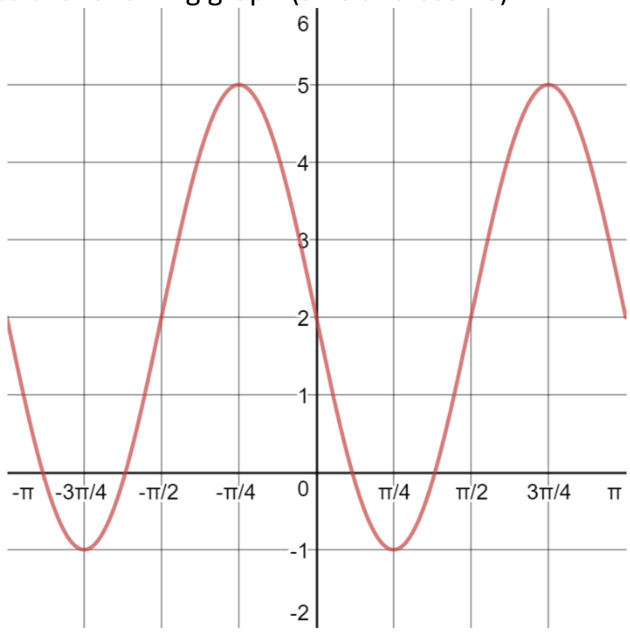
$$4\cos\left(\frac{\pi}{7}x\right) - 1$$

Graphing these functions is a bit easier since all the needed information is right in front of you on your equation.

Big Ideas you need to know and understand:
Graphing complex trig functions is just applying transformations of the basic sine and cosine graph.
There is no one equation that will match the graph of a trig function. Since they are periodic there will be infinitely many equations that work plus sine and cosine varieties.

Review questions:

3. What is the equation to the following graph (sine and cosine)?



4. What is the equation (sine and cosine) to a trig function that has an amplitude of 2 and passes through the midline at the points $(4, -3)$ and $(10, -3)$ (consecutive midline points)?
5. Graph the function $1.5\sin\left(\frac{\pi}{4}(x-3)\right)+0.5$

Solutions:

3. $-3\sin(2x)+2$ OR $3\cos\big(2(x+\pi/4)\big)+2$
4. $\pm 2\sin\left(\frac{\pi}{6}(x+2)\right)-3$ OR $\pm 2\cos\left(\frac{\pi}{6}(x-1)\right)-3$ (plus minus because we don't know if it starts going up or going down first.)
5. Use Desmos to check.

Building Equations

When we are building an equation to model something periodic, we work to abstract the problem to find its basic characteristics.

- The period is the length of 1 full cycle. Typically, the period is a measurement of time (things change periodically over time), but it doesn't have to be (maybe period is a measure of distance or something ese).
- The amplitude will be half of the max minus the min, just like with points. It will be measured in units of your output (what your equation is modelling, ie height, temperature, etc).
- The vertical displacement is the average value of the output (what your equation is modelling, ie height, temperature, etc).
- To determine the phase shift, you need to consider the initial conditions given to you in the problem. Sometimes there are no initial conditions and you can choose no phase shift, but often you will need to match the constraints of the problem.

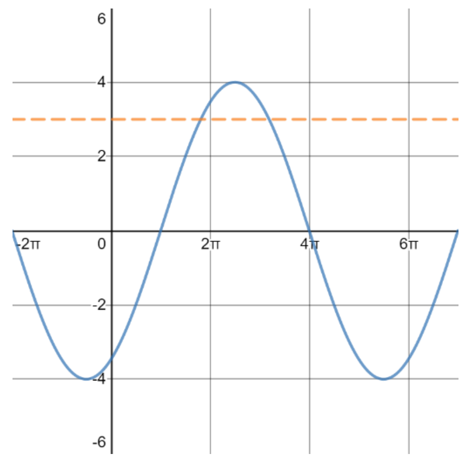
Big Ideas you need to know and understand:
Solving by graphing is fine in most instances to get an approximate solution. We can use algebra to solve these equations sometimes; however, this is not always possible, nor is it required in most everyday problems.
Building equations from word problems requires the same steps as the previous section.

Review questions:

6. What is the general solution to the following equation:
$$4\sin\left(\frac{1}{3}(x-\pi)\right)=3$$
7. A Ferris wheel has a radius of 15m and picks up passengers 3m from the ground and then moves up first. The wheel makes one full rotation every 90 seconds. If the highest point passengers reach is 32m above the ground after 40 seconds. Determine an equation for the height of the passenger at time t. What times in the first 2 minutes will the passenger be 10m above the ground?

Solutions:

6. The graph should look like:



My quick guess is that $x \approx 5, 10 + 6\pi n, n \in \mathbb{Z}$

Solving it exact we get
$$\frac{x-\pi}{3}=0.8, \qquad 2.3+2\pi n$$
$$x = 5.685\ldots, \qquad 10.022\ldots + 6\pi n$$

7. $h(t) = 15\cos\left(\frac{2\pi}{90}(t-40)\right)+17$

If $h(t)=10$, then $\frac{2\pi}{90}(t-40) = -2, 2+2\pi n, n \in \mathbb{Z}$. Solving for t we get
$$t = 11, 69 + 90n$$
So when $t \le 120$ we get $t = 11, 69, 101$ seconds