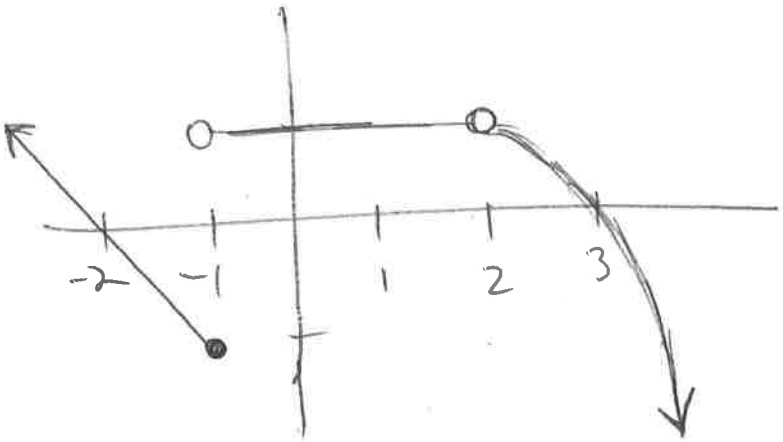
Composition of Function Worksheet

1. Graph the function

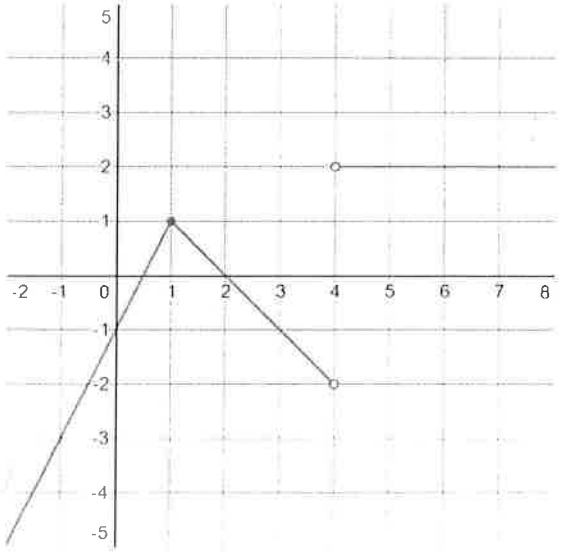
$$f(x) = \begin{cases} -x-2, & x \le -1 \\ 1, & -1 < x < 2 \\ -(x-2)^2 + 1, & x > 2 \end{cases}$$



2. Does $f^{-1}(x)$ exist? If yes determine an expression for it. If no say why not and restrict the domain so that it does.

not in general b/c $f(-2) = f(3) = 0$ (fails horizontal line test) $f^{-1}(x)$ exists for $x > 2$ and $f^{-1}(x)$ exists for $x \le -1$

3. Determine the function to the following graph



$$f(x) = \begin{cases} 2x-1, & x \le 1 \\ -x+2, & 1 < x < 4 \\ 2, & x > 4 \end{cases}$$

4. Consider the following functions
   i. $f(x) = A$
   ii. $g(x) = Bx$
   iii. $h(x) = Cx^2$
   iv. $j(x) = Dx^3$

a. For each function simplify:

$$\frac{f(x+\varepsilon) - f(x)}{\varepsilon}$$

i) $\frac{f(x+\varepsilon) - f(x)}{\varepsilon} = \frac{A - A}{\varepsilon} = 0$

ii) $\frac{g(x+\varepsilon) - g(x)}{\varepsilon} = \frac{B(x+\varepsilon) - Bx}{\varepsilon}$

$= B$

iii) $\frac{h(x+\varepsilon) - h(x)}{\varepsilon} = \frac{C(x+\varepsilon)^2 - Cx^2}{\varepsilon}$

$= \frac{Cx^2 + 2Cx\varepsilon + C\varepsilon^2 - Cx^2}{\varepsilon}$

$= 2Cx + C\varepsilon$

iv) $\frac{j(x+\varepsilon) - j(x)}{\varepsilon} = \frac{D(x+\varepsilon)^3 - Dx^3}{\varepsilon}$

$= \frac{Dx^3 + 3Dx^2\varepsilon + 3Dx\varepsilon^2 + D\varepsilon^3 - Dx^3}{\varepsilon}$

$= 3Dx^2 + 3Dx\varepsilon + D\varepsilon^2$

b. If we continue the pattern, what do you think the simplified form would be for the function Ex^4? Justify yourself.

$$\frac{E(x+\varepsilon)^4 - Ex^4}{\varepsilon} \approx 4Ex^3 + \underline{?}\,Ex^2\varepsilon + \underline{?}\,Ex\varepsilon^2 + \underline{?}\,E\varepsilon^3$$

The first coefficient goes up by 1 and the power goes up by 1. The rest of it will have $Ex^n\varepsilon^m$ where $n+m=3$. The coefficients will be something but fixed. The last one will be 1 though b/c its just $E(\varepsilon)^4$

c. Using evidence, what do you think the simplified form of x^n would be?

$$\frac{(x+\varepsilon)^n - x^n}{\varepsilon} \approx nx^{n-1} + \_\_x^{n-2}\varepsilon + \ldots + \varepsilon^{n-1}$$

There will be no x^n b/c $(x+\varepsilon)^n = x^n + \ldots$ so they will cancel. Then generalize the pattern on the previous example.