5.2 DEFINITE INTEGRALS

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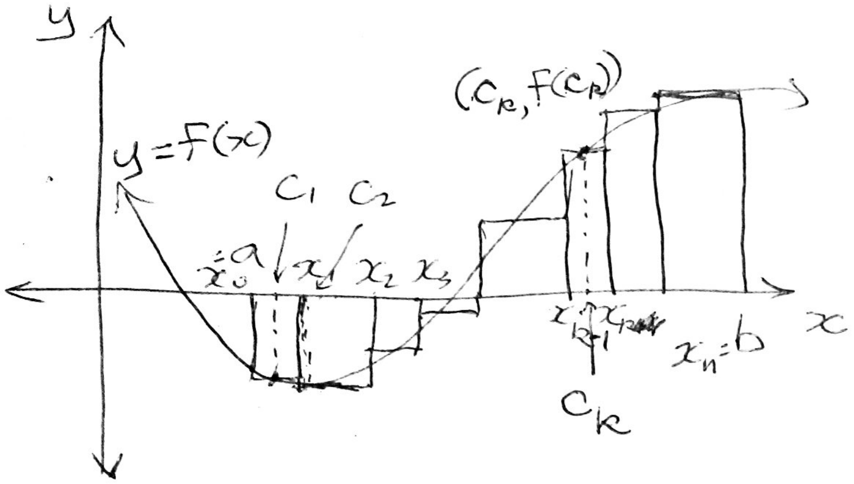

$$a < x_1 < x_2 < \cdots < x_{k-1} < x_k < \cdots < x_{n-1} < b$$

($a = x_0$, $b = x_n$)

Partition $P = \{x_0, x_1, \ldots, x_n\}$
(A Set of x values on interval $[a, b]$)
There are n subintervals ~~that~~ of ~~can have~~ varying lengths.

c_k is in kth subinterval
$f(c_k)$ is the height of rectangle from the x-axis

Area of that rectangle $= f(c_k) \cdot \Delta x_k$

Sum of all rectangle areas

$$\underbrace{S_n = \sum_{k=1}^{n} f(c_k) \cdot \Delta x_k}_{\text{An example of a Riemann Sum.}}$$

DEFINITION: The Definite Integral as a Riemann Sum

$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} f(c_k) \cdot \Delta x_k = I$$

- Function f is defined on interval $[a,b]$
- c_k is chosen arbitrarily on subinterval $[x_{k-1}, x_k]$
- If there exists a number I such that

then f is integrable on $[a, b]$ and I is the definite integral of f over $[a,b]$

THEOREM 1 The Existance of Definite Integrals

All continuous functions are integrable.

(If function f is continuous on interval $[a, b]$, then its definite integral exists.)

THE DEFINITE INTEGRAL OF A CONTINUOUS FUNCTION ON $[a, b]$.

Let ... f be continuous on $[a,b]$,
... $[a,b]$ be partitioned into subintervals of equal length $\Delta x = \frac{(b-a)}{n}$

The definite integral of f over $[a,b]$ is

$$\lim_{n \to \infty} \sum_{k=1}^{n} f(c_k) \Delta x$$

where c_k is chosen arbitrarily in the kth subinterval.

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$$\boxed{\text{Recall} \quad \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}}$$

Upper limit of integration   Integrand

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\Delta x = \underbrace{\int_a^b f(x)\,dx}_{\text{Integral of } f \text{ from } a \text{ to } b}$$

variable of integration

Integral sign   Lower limit of integration

If variable is t

$$\int_a^b f(t)\,dt$$

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$$\underbrace{\lim_{\|P\|\to 0} \sum_{k=1}^{n} c_k^2 \Delta x_k}$$

P is any partition of [0,2]

$$= \int_0^2 x^2\,dx$$

#4 $$\lim_{\|P\|\to 0} \sum_{k=1}^{n} \frac{1}{1-c_k}\Delta x_k = \int_2^3 \frac{1}{1-x}\,dx$$

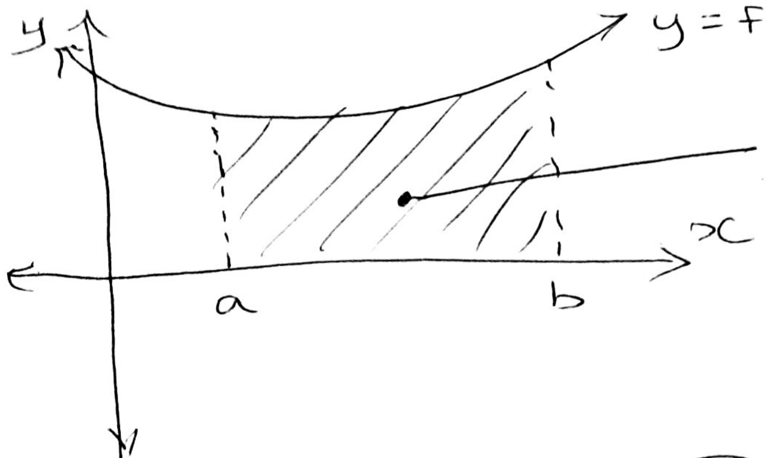
where P is any partition of [2,3]

DEFINITE ~~ARE~~ INTEGRAL AND AREA

Riemann sum $\sum f(c_k)\Delta x_k$ is an estimate of the area between the curve and the x-axis from a to b.

DEFINITION Area of a Curve (as a Definite Integral)

IF $y=f(x)$ is non negative and integrable on closed ~~area~~ interval $[a,b]$ then the area under the curve $y=f(x)$ from a to b is the integral of f from ~~A~~ to b $A=\int_a^b f(x)\,dx$.

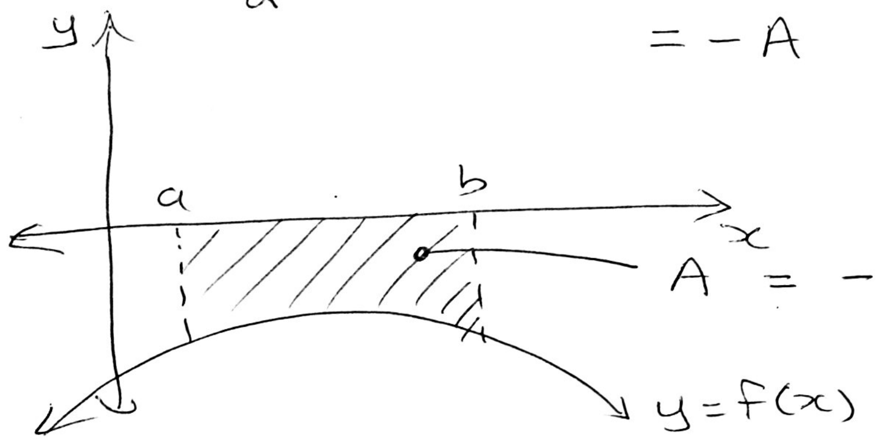


$A = \int_a^b f(x)\,dx$.

And if $f(x)$ is non positive?

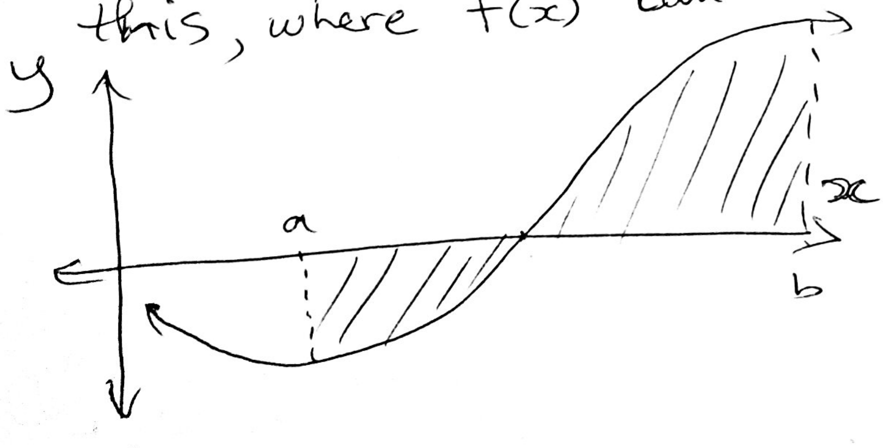
The non-zero terms $f(c_k)\Delta x_k$ in the Riemann sums for f over interval $[a,b]$ are negatives of rectangle areas,

$$\int_a^b f(x)\,dx = -\,(\text{the area}) \quad \text{if } f(x) \le 0$$
$$= -A$$



$A = -\int_a^b f(x)\,dx$

<u>Implication</u> In a situation like this, where $f(x)$ can be both negative and positive on interval $[a, b]$

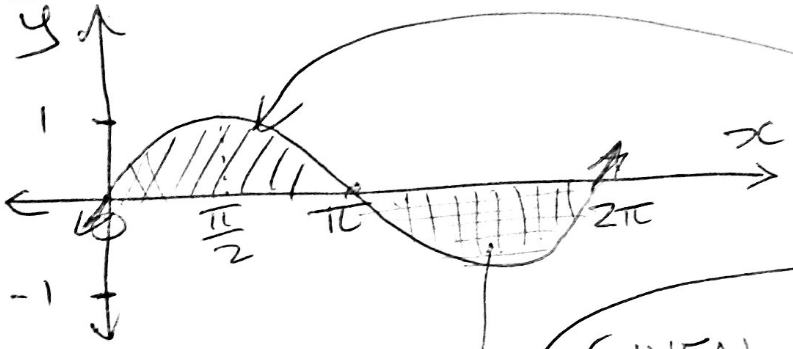


$\int_a^b f(x)\,dx <$ ~~less than~~ the total area between the curve and ~~of~~ the x-axis

For any integrable function,

$$\int_a^b f(x)\,dx = (\text{area above the } x\text{-axis}) - (\text{area below the } x\text{-axis})$$

EXPLORATION 1 [P.264]



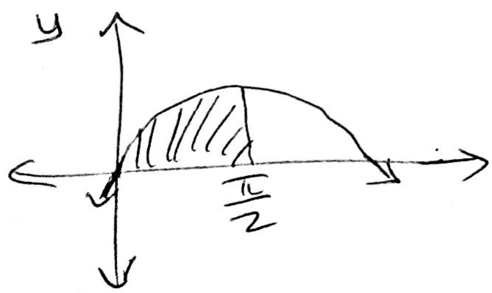
GIVEN $\int_0^{\pi} \sin x\,dx = 2$

1. $\int_{\pi}^{2\pi} \sin x\,dx = \boxed{-2}$

2. $\int_0^{2\pi} \sin x\,dx = \int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} \sin x\,dx$

$$= 2 + (-2)$$
$$= \boxed{0}$$

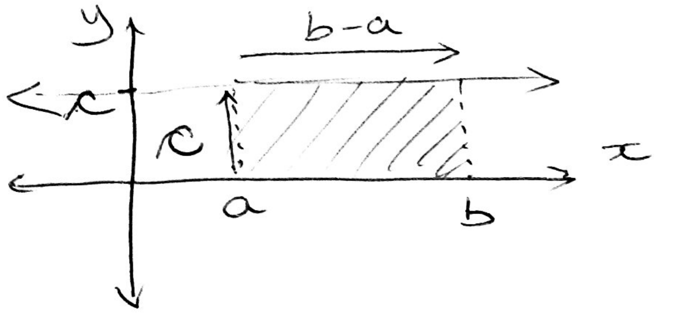
3. $\int_0^{\frac{\pi}{2}} \sin x\,dx = \boxed{1}$

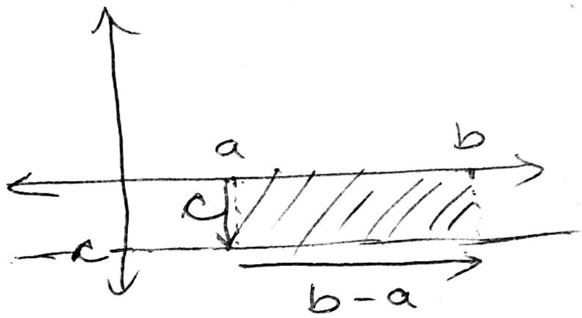


THEOREM 2 The Integral of a Constant

If $f(x) = C$, where C is a constant, and f is on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = \int_a^b c\,dx = c(b-a)$$





$c(b-a) > 0$

$c(b-a) < 0$

PROOF

$\sum_{k=1}^{n} f($