Derivative of a Product and Quotient

Goal: • Can determine the derivative of a product and quotient of functions • Understands how limits are used to define these rules and can intuit a
Terminology: • Product Rule • Quotient Rule

Review: Determine $\frac{dA}{dt}$ given that $A = (12 + 1.5t)(156 + 13t)$.

Rather than having to do the long and often tedious expansion, we ask ourselves can we determine the general derivative of a product?

$$\frac{d}{dx}(f+g) = (f+g)' = ???$$

$$\frac{d}{dx}(f+g) = (f+g)' = \lim_{h\to 0}\frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= g(x)\cdot\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} + f(x)\cdot\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$
$$= g\frac{df}{dx} + f\frac{dg}{dx} = f'g + g'f$$

But why would you think to do this??

Example: Given $y = (4x^2 - 7x + 1)(3x^4 + x^3 - 2x)$ determine $\frac{dy}{dx}$.

The quotient rule has a similar style argument and is defined as

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

Example: Determine y' given that $y = \frac{3r^4 - 2r}{r + 4r^2}$

Practice Problems: 3.3: # 9, 10, 13-19, 22-24, 33

 # 39, 40

In Class Evidence

10. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ given that

$$y = \frac{1}{4x^4} - \frac{1}{3x^3} + \frac{1}{2x^2} - \frac{1}{x} + 1$$

19. Do not expands. Find, $\frac{dy}{dx}$ given that

$$y = \frac{(x+1)(x+2)}{(x-1)(x-2)}$$

39. The members of the Blue Boar society always divide the pavilion rental fee for their picnics equally among the members. Currently there are 65 members and the pavilion rents for $250. The pavilion cost is increasing at a rate of $10 per year, while the Blue Boar membership is increasing at a rate of 6 members per year. What is the current rate of change in each members share of the pavilion fee?

40. Create a geometric argument why the quotient rule is true.