Using Desmos and Geogebra with Compositions and Translations

Part 1: Compositions:

Consider the functions:

$$f(x) = 2x + 4, x \in [-3, 1] \qquad g(x) = \sqrt{x + 1}$$

Use mapping notation to identify the domain and range of each function and then determine the domain and range of $g \circ f$. Check using technology by defining both functions and graphing $g\big(f(x)\big)$

Do the same to determine the domain and range of $f \circ g$

Consider the functions:

$$p(x) = \begin{cases} 1 - 2x, -2 \le x < 0 \\ x + 1, \qquad 0 \le x < 2 \end{cases} \qquad\qquad q(x) = \frac{1}{x}$$

Use mapping notation to identify the domain and range of each function and then determine the domain and range of $p \circ q$. Check using technology by defining both functions and graphing $p\big(q(x)\big)$

Do the same to determine the domain and range of $q \circ p$

Part 2: Characteristics and Translations

We want to consider how vertical and horizontal translations effect the following characteristics of functions:

- Domain
- Range (included in this is max and min values)
- Zeros
- y-intercept
- Vertical asymptotes
- Horizontal asymptotes

Domain and Range

Consider the function $f(x) = x^2, -1 \leq x \leq 2$. Graph this function and graph a box around it to highlight the domain and range. To do this type the following into Desmos

$$x = [-1,2]\{0 \leq y \leq 4\}$$
$$y = [0,4]\{-1 \leq x \leq 2\}$$

The square brackets tell Desmos to make a list and then we are just graphing all those lines.

Add a function $g(x) = f(x - c) + d$ to shift the function left/right and up/down

Add new vertical and horizontal lines as functions of c and d and explain how shifting the function changes the domain and range in a predictable way.

Zeros and y-intercept:

Consider the function $p(x) = x^3 + 3x^2 - x - 3$. Graph the function and plot the zeros and y-intercept on the graph by plotting the coordinates

$$(-3, 0), (-1, 0), (1, 0)$$
$$(0, -3)$$

Add a function q that translates the function left/right and up/down.

Add new points for the zeros and intercept as functions of the shifts.

Explain how shifting the function changes the zeros and changes the y-intercept in a predictable way.

Asymptotes:

Consider the function $R(x) = \frac{x^2+1}{x^2+x-3}$
Graph the function and graph the asymptotes at $x = -3$, $x = 2$, and $y = 1$

Add a function T that translates the function left/right and up/down.

Add new lines for the asymptotes as functions of the shifts. Explain how shifting the function changes the asymptotes in a predictable way.