Using Desmos and Geogebra with Compositions and Translations

Part 1: Compositions:

Consider the functions:

$$f(x) = 2x + 4, x \in [-3, 1] \qquad g(x) = \sqrt{x+1}$$

Use mapping notation to identify the domain and range of each function and then determine the domain and range of $g \circ f$. Check using technology by defining both functions and graphing $g(f(x))$

$f: [-3,1] \to [-2,6]$ $\quad$ $g: [-1,\infty) \to [0,\infty)$

intersection $= [-1,6]$

Domain

if $x \in$ Domain $\Rightarrow f(x) \in [-1,6]$

$\Rightarrow -1 \le 2x+4 \le 6$

$-5 \le 2x \le 2$

$-2.5 \le x \le 1$

$\Rightarrow$ Domain is $[-2.5, 1]$

Range

if $x \in [-1,6]$ then $g(x) \in$ Range

$-1 \le x \le 6$

$\Rightarrow 0 \le x+1 \le 7$

$\Rightarrow 0 \le \sqrt{x+1} \le \sqrt{7}$

$\Rightarrow 0 \le g(x) \le \sqrt{7}$

$\Rightarrow$ Range is $[0, \sqrt{7}]$

Do the same to determine the domain and range of $f \circ g$

$g: [-1,\infty) \to [0,\infty)$ $\quad$ $f: [-3,1] \to [-2,6]$

intersection is $[0,1]$

Domain

if $x \in$ Domain $\Rightarrow g(x) \in [0,1]$

$\Rightarrow 0 \le \sqrt{x+1} \le 1$

$0 \le x+1 \le 1$

$-1 \le x \le 0$

$\Rightarrow$ Domain $= [-1, 0]$

Range

if $x \in [0,1] \Rightarrow f(x) \in$ Range

$0 \le x \le 1$

$0 \le 2x \le 2$

$4 \le 2x+4 \le 6$

$\Rightarrow$ Range $= [4, 6]$

Consider the functions:

$$p(x) = \begin{cases} 1 - 2x, -2 \le x < 0 \\ x + 1, \qquad 0 \le x < 2 \end{cases} \qquad\qquad q(x) = \frac{1}{x}$$

Use mapping notation to identify the domain and range of each function and then determine the domain and range of $p \circ q$. Check using technology by defining both functions and graphing $p(q(x))$

$q: \mathbb{R}\setminus\{0\} \rightarrow \mathbb{R}\setminus\{0\}$ $\qquad$ $p: [-2,2] \rightarrow [1,5]$

☆ $\mathbb{R}\setminus\{0\} = \{x \mid x \neq 0\}$, the symbol $\setminus$ means set minus.

intersection $= [-2,0) \cup (0,2] = [-2,2]\setminus\{0\}$

Domain

if $x \in$ Domain $\Rightarrow q(x) \in [-2,2]\setminus\{0\}$

$\Rightarrow -2 \le \frac{1}{x} < 0$ or $0 < \frac{1}{x} \le 2$

$\Rightarrow -2x \ge 1 > 0$ or $0 < 1 \le 2x$

$\Rightarrow x \le -\frac{1}{2} < 0$ or $0 < \frac{1}{2} \le x$

$\Rightarrow x \le -\frac{1}{2}$ or $x \ge \frac{1}{2}$

$\Rightarrow$ Domain $= (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, \infty)$

Range

if $x \in [-2,2]\setminus\{0\} \Rightarrow p(x) \in$ Range

$-2 \le x < 0$ or $0 < x \le 2$

$\Rightarrow 4 \ge -2x > 0$ or $1 < x+1 \le 3$

$5 \ge 1-2x > 1$

$\Rightarrow p(x) \in (1,5]$ or $p(x) \in (1,3]$

$\Rightarrow$ Range $= (1,5]$

Do the same to determine the domain and range of $q \circ p$

$p: [-2,2] \rightarrow [1,5]$ $\qquad$ $q: \mathbb{R}\setminus\{0\} \rightarrow \mathbb{R}\setminus\{0\}$

intersection $= [1,5]$

if $x \in$ Domain $\Rightarrow p(x) \in [1,5]$

$\Rightarrow$ Domain $= [-2,2]$

if $x \in [1,5] \Rightarrow q(x) \in$ Range

$1 \le x \le 5$

$\Rightarrow 1 \ge \frac{1}{x} \ge \frac{1}{5}$

Range $= [\frac{1}{5}, 1]$

Part 2: Characteristics and Translations

We want to consider how vertical and horizontal translations effect the following characteristics of functions:

- Domain
- Range (included in this is max and min values)
- Zeros
- y-intercept
- Vertical asymptotes
- Horizontal asymptotes

Domain and Range

Consider the function $f(x) = x^2, -1 \leq x \leq 2$. Graph this function and graph a box around it to highlight the domain and range. To do this type the following into Desmos

$$x = [-1,2]\{0 \leq y \leq 4\}$$
$$y = [0,4]\{-1 \leq x \leq 2\}$$

The square brackets tell Desmos to make a list and then we are just graphing all those lines.

Add a function $g(x) = f(x - c) + d$ to shift the function left/right and up/down

Add new vertical and horizontal lines as functions of c and d and explain how shifting the function changes the domain and range in a predictable way.

Shifting left/right changes the domain

$\Rightarrow$ new domain is $[-1+c, 2+c]$

shifting up/down changes the range

$\Rightarrow$ New range is $[0+d, 4+d]$

Zeros and y-intercept:

Consider the function $p(x) = x^3 + 3x^2 - x - 3$. Graph the function and plot the zeros and y-intercept on the graph by plotting the coordinates

$$(-3, 0), (-1, 0), (1, 0)$$
$$(0, -3)$$

Add a function q that translates the function left/right and up/down.

Add new points for the zeros and intercept as functions of the shifts.

Explain how shifting the function changes the zeros and changes the y-intercept in a predictable way.

Since the points move $(x,y) \mapsto (x+c, y+d)$

for a zero to remain a zero we need there

to be NO vertical shift

$(-3,0) \mapsto (-3+c, 0)$ etc

The same is true for y-intercepts as there can

be NO horizontal shift

$(0,-3) \mapsto (0, -3+d)$

Asymptotes:

Consider the function $R(x) = \frac{x^2+1}{x^2+x-\cancel{8}6}$ oops!

Graph the function and graph the asymptotes at $x = -3$, $x = 2$, and $y = 1$

Add a function T that translates the function left/right and up/down.

Add new lines for the asymptotes as functions of the shifts. Explain how shifting the function changes the asymptotes in a predictable way.

The asymptotes change as the domain + range change

shifting left/right moves the vertical asymptotes

↳ new at $x = -3 + c$ and $x = 2 + c$

shifting up/down moves the horizontal asymptotes

↳ new at $y = 1 + d$