Antiderivatives and Solving Differential Equations

Goal:
- Can find the antiderivative of polynomials, radicals, exponential, and simple rational functions.
- Can use antiderivatives to solve differential equations and use initial values to find the exact solution

Terminology:
- Antiderivative and $\int$
- Differential equation
- Initial value

Discussion question: What does the following symbol mean and do?

$$\frac{d}{dx}$$

From the discussion it's natural to want to go backwards and solve the following problem. Given a known $f(x)$, determine $F(x)$ such that

$$\frac{d}{dx}\underbrace{\boxed{\quad ? \quad}}_{F(x)} = \underbrace{f(x)}_{\text{known}}$$

Example: Determine $F(x)$ such that

$$\frac{d}{dx}F(x) = \sqrt{x} + 3x^4$$

** Remember that this is reversible so check!

Example: Determine $F(x)$ if

$$F'(x) = \frac{1}{x-3} + e^{-2x}$$

Practice: Determine $F(x)$ such that

$$\frac{d}{dx}F(x) = \sqrt[3]{1-5x} + x^{-\frac{3}{2}}$$

Practice: Determine $F(x)$ such that

$$F'(x) = \frac{1}{x-4} + \frac{5}{1-2x} + 2^{x}$$

Common antiderivatives we need to memorize

$g(x)$	$g'(x) = \frac{d}{dx}g(x)$		$f(x)$	$F(x) = \int f(x)$
x^n				
e^x				
$\ln x$				
C				

Since there is a constant C in every general antiderivative, when we solve differential equations we have solved them up to some vertical shift factor. We need to know some value of the function to find the exact solution. Hence, an **initial value** to **initialize** the solution.

Example: Solve for y given $y(0) = 2$

$$\frac{dy}{dx} = e^x + \frac{2}{x-1}$$

Example: Solve for V given $V(1) = 1$

$$\frac{dV}{dt} = \sqrt{1+3t} - 1$$

Example: If F is tangent to the line $y = 3x$ and $F'(x) = x^2 - 1$ then determine F

Practice: Determine G given $G(-1) = 1$ and

$$G'(x) = 2x + \frac{1}{3 - x}$$

Practice: If the function H is tangent to the curve $x + y = 1$ and $H'(x) = -e^{-2x}$ then what is H?

Practice Problems: 9.1: # 1-3 & 5 (do what you need), 6
9.2: # 1, 2abd, 3 & 4bcd (do what you need), 5

9.1 # 7acd; 9.2 # 7